Sequences and Series

Sequence term: 1, 3, 5, 7, 9, 11 $\rightarrow$ Collection of particular numbers in particular pattern.

Series $=$ $1+3+5+7+9+11$ $\rightarrow$ representation of numbers of sequence with '+' sign

Sum of Series $=$ 36

Finite sequence $\rightarrow$ no. of terms = Finite $(1, 2, 3, 4 \dots, 100)$

Infinite Sequence $\rightarrow$ no. of terms = infinite $(1, 2, 3, \dots)$

Sequence $\rightarrow$ $a_1, a_2, a_3, a_4 \dots, a_n, a_{n+1}, a_{n+2} \dots$

Series $\rightarrow$ $a_1 + a_2 + a_3 + \dots + (a_n) + (a_{n+1}) + (a_{n+2}) + \dots a_{1000}$

$\rightarrow \sum_{r=1}^{1000} a_r$

n^{th} term $\rightarrow$ General Term = Formula

Exercise **8.1**

$\boxed{Q.1}$ $a_n = n.(n+2)$

$a_1 = 1.(1+2) = 1\times3 = 3$

$a_2 = 2(2+2) = 2\times4 = 8$

$a_3 = 3.(3+2) = 15$

$a_4 = 4.(4+2) = 24$

$a_5 = 5.(5+2) = 35$

$3, 8, 15, 24, 35, \ldots$

$\boxed{Q.2}$ $a_n = \frac{n}{n+1}$

$a_1 = \frac{1}{1+1} = \frac{1}{2}$

$a_2 = \frac{2}{2+1} = \frac{2}{3}$

$a_3 = \frac{3}{3+1} = \frac{3}{4}$

$a_4 = \frac{4}{4+1} = \frac{4}{5}$

$a_5 = \frac{5}{5+1} = 6$

$\boxed{Q.3}$ $a_n = 2^n$

$a_1 = 2^1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$

$a_5 = 2^5 = 32$

$\boxed{Q.4}$ $a_n = \frac{2n-3}{6}$

$a_1 = \frac{2\times1-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$

$a_2 = \frac{2\times2-3}{6} = \frac{4-3}{6} = \frac{1}{6}$

$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$

$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$

$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$

$\boxed{Q.5}$ $a_n = (-1)^{n-1} \cdot 5^{n+1}$

$a_1 = (-1)^{1-1} \cdot 5^{1+1} = (-1)^0 \cdot 5^2 = 1\times25 = 25$

$a_2 = (-1)^{2-1} \cdot 5^{2+1} = -125$

$a_3 = (-1)^{3-1} \cdot 5^{3+1} = +625$

$a_4 = (-1)^{4-1} \cdot 5^{4+1} = -3125$

$a_5 = (-1)^{5-1} \cdot 5^{5+1} = +15625$

$\boxed{Q.6}$ $a_n = n.\left(\frac{n^2+5}{4}\right)$

$a_1 = 1.\left(\frac{1^2+5}{4}\right) = 3/2$

$a_2 = 2.\left(\frac{2^2+5}{4}\right) = 9/2$

$a_3 = 3.\left(\frac{3^2+5}{4}\right) = \frac{21}{2}$

$a_4 = 4.\left(\frac{4^2+5}{4}\right) = 21$

$a_5 = 5.\left(\frac{5^2+5}{4}\right)$

$= \frac{75}{2}$

Q.7

$a_n = 4n - 3$

$a_{17} = 4(17) - 3 = 68 - 3 = 65$ ✓

$a_{24} = 4(24) - 3 = 96 - 3 = 93$ ✓

Q.8. $a_n = \frac{n^2}{2^n}$

$a_7 = \frac{(7^2)}{2^7} = \frac{49}{128}$ ✓

Q.9 $a_n = (-1)^{n-1} \cdot n^3$

✓ $a_9 = (-1)^{9-1} \cdot 9^3$

$= (-1)^8 \cdot 729$

$= +729$ ✓

Q.10 $a_n = \frac{n(n-2)}{n+3}$

✓ $a_{20} = \frac{20 \times (20-2)}{20+3}$

$= \frac{20 \times 18}{23} = \frac{360}{23}$ ✓

Q.11 $a_1 = 3$, $\boxed{a_n = 3(a_{n-1}) + 2}$, $n > 1$

$a_1 = 3$
$a_2 = 11$
$a_3 = 35$

$a_n = 3 \cdot (a_{n-1}) + 2$

$\boxed{n=2}$ $a_2 = 3 \times a_{2-1} + 2 = 3 \times a_1 + 2 = 3 \times 3 + 2 = 11$

$\boxed{n=3}$ $a_3 = 3 \cdot (a_{3-1}) + 2 = 3 \times a_2 + 2 = 3 \times 11 + 2 = 35$

$n=4$ $a_4 = 3 \cdot a_3 + 2$

$= 3 \times 35 + 2 = 107$

$a_5 = 3 \times a_4 + 2$

$= 3 \times 107 + 2 = 323$

Series: $3 + 11 + 35 + 107 + 323 + \ldots$

$\boxed{Q.12}$ $a_1 = -1$

$$a_n = \frac{(a_{n-1})}{n}$$

n=2 $a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-1}{2}$

$$a_3 = \frac{a_2}{3} = \frac{\left(-\frac{1}{2}\right)}{3} = -\frac{1}{6}$$

$$a_4 = (a_3)/4 = \frac{\left(-\frac{1}{6}\right)}{4} = -\frac{1}{24}$$

$$a_5 = \frac{a_4}{5} = \frac{\left(-\frac{1}{24}\right)}{5} = \frac{-1}{120}$$

Series $= (-1) + \left(\frac{-1}{2}\right) + \left(-\frac{1}{6}\right) + \left(\frac{-1}{24}\right) + \left(\frac{-1}{120}\right) + \ldots$

$\boxed{Q.13}$ $a_1 = a_2 = 2$

$a_n = a_{n-1} - 1$, $\underline{n > 2}$

n=3 $a_3 = a_{3-1} - 1 = a_2 - 1 = 2 - 1 = 1$

n=4 $a_4 = a_3 - 1 = 1 - 1 = 0$

n=5 $a_5 = a_4 - 1 = 0 - 1 = -1$

Series $= 2 + 2 + 1 + 0 + (-1) + \ldots$

Q.14 Fibonacci Sequence $a_1 = a_2 = 1$

$$a_n = a_{n-1} + a_{n-2}, \quad n > 2$$

$n=3$, $a_3 = a_2 + a_1 = 1 + 1 = 2$

$n=4$, $a_4 = a_3 + a_2 = 2 + 1 = 3$

$n=5$, $a_5 = a_4 + a_3 = 3 + 2 = 5$

$n=6$, $a_6 = a_5 + a_4 = 5 + 3 = 8$

$\dfrac{a_{n+1}}{a_n} = ?$ $n = 1, 2, 3, 4, 5$

$n=1$, $\dfrac{a_2}{a_1} = \dfrac{1}{1} = 1$ ✓

$n=2$, $\dfrac{a_3}{a_2} = \dfrac{2}{1} = 2$ ✓

$n=3$, $\dfrac{a_4}{a_3} = \dfrac{3}{2}$ ✓

$n=4$, $\dfrac{a_5}{a_4} = \dfrac{5}{3}$ ✓

$n=5$, $\dfrac{a_6}{a_5} = \dfrac{8}{5}$ ✓

Arithmetic Progression (A.P.) $\Rightarrow$

$$\boxed{a_n = a_{n-1} + d}$$

$d \rightarrow$ Fixed no.

Common Difference
(C.D.)

Let First term $= a$

$CD = d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

AP $\rightarrow$ $a,\ a+d,\ a+2d,\ a+3d,\ \ldots\ldots,\ \underline{a+(n-1).d}$

$\uparrow a_1\ (t_1)$ $\quad \uparrow a_2$ $\quad \uparrow a_3$ $\quad \uparrow a_4$ $\quad \uparrow$ General term $(a_n) = n^{th}$ term

<u>General Term</u> $\quad a_n = n^{th}$ term $= a+(n-1).d$

Sum of AP. $= S_n = a_1 + a_2 + a_3 + \ldots + a_n = \frac{n}{2}\left(2a+(n-1).d\right)$

$S_n = \frac{n}{2}(a + l) = \frac{n}{2}(a_1 + a_n)$

$\uparrow$ last term

$a_n = (S_n) - (S_{n-1})$

$$\begin{pmatrix} \cancel{a_1} \\ \cancel{a_2} \\ \vdots \\ \cancel{a_{n-1}} \\ a_n \end{pmatrix} - \begin{pmatrix} \cancel{a_1} \\ \cancel{a_2} \\ \cancel{a_{n-1}} \end{pmatrix}$$

Properties:

① If a, b, c are in AP, then $\boxed{b = \frac{a+c}{2}}$ or $\boxed{2b = a+c}$

$b - a = c - b$

② $AP \to +K$, $-K$, $\times K$, $\div K$ constant (for each term) of A.P. $\to$ new sequence / ~~new~~ Resultant sequence $\to$ New AP.

$AP_1 + AP_2 = AP_3$

$a_1 + b_1 = c_1$

$a_2 + b_2 = c_2$

× a-3d a-2d a-d ~~a~~ a+d a+2d a+3d ×

③ Assumption of terms in AP. (Condition → their Sum is given).

Assume 3 - numbers: $a-d,\ a,\ a+d$ (CD = d)

Assume 4 - numbers: $\boxed{a-3d,\ a-d,\ a+d,\ a+3d}$ CD = 2d

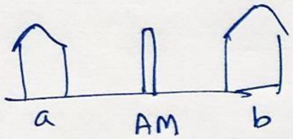
Assume 5 - numbers: CD = d: $a-2d,\ a-d,\ a,\ a+d,\ a+2d$

Assume 6 - numbers: (C.D = 2d): $a-5d,\ a-3d,\ a-d,\ a+d,\ a+3d,\ a+5d$

Arithmetic Mean (AM)

(1) AM of two no. a & b $= \frac{a+b}{2}$

$$\boxed{AM = \frac{a+b}{2}}$$



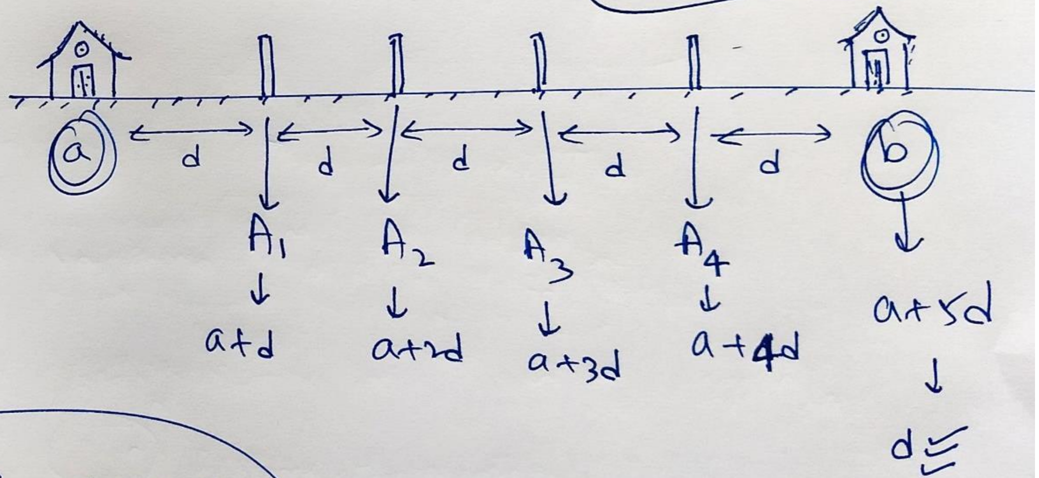
AM of a, b, c, d

$AM = \frac{a+b+c+d}{4}$

(2) n- AMs → find → between a & b

$A_1, A_2, A_3, \ldots A_n,$

only one unknown = d



So that

$$\boxed{a, A_1, A_2, A_3, \ldots, A_n, b}$$ → form AP

a

$A_1 = a+d$

$A_2 = a+2d$

$\vdots$

$A_n = a + n.d$

$b = a+(n+1).d$

Hint

$b = a+(n+1).d$

$b-a = (n+1).d$

$\Rightarrow \boxed{\frac{b-a}{n+1} = d}$

Exercise 8.2 → A.P.

① nth term $= a_n = a+(n-1).d$ ← $d = C.D.$

② $a_n = S_n - S_{n-1}$

③ $S_n = \frac{n}{2}(2a+(n-1).d) = a_1+a_2+\dots+a_n$

④ $S_n = \frac{n}{2}(a+l) = \frac{n}{2}(a_1+a_n)$

Q.1 Sum of odd integers from '1' to '2001'

$S_n = 1+3+5+7+\dots+1999+2001$

$n=?$

$a_1 = a$

$l = a_n = a+(n-1).d$

$d = \text{Common Difference} = 2$

$a_n = a+(n-1).d$

$\Rightarrow 2001 = 1+(n-1).2$

$\Rightarrow 2000 = (n-1).2$

$\Rightarrow 1000 = n-1$

$\Rightarrow 1001 = n$

$S_n = S_{1001} = \frac{n}{2}(a+l)$

$S_{1001} = \frac{1001}{2}(1+2001)$

$= \frac{1001}{\cancel{2}} \times \cancel{2002}^{1001}$

$= 1001 \times 1001$

$= 1002001$

Q.2 Sum of all natural no. lying b/w 100 & 1000 which are multiple of '5'

$$S_n = 105 + 110 + 115 + \cdots + 990 + 995$$

$a_1 = a$, a_2, a_{n-1}

$d = 5$

$a_n = 995$

$\Rightarrow a + (n-1).d = 995$

$\Rightarrow 105 + (n-1).5 = 995$

$\Rightarrow (n-1).\cancel{5} = \cancel{890}\ 178$

$\Rightarrow n-1 = 178$

$n = 179$

$$S_n = \frac{n}{2}(a + l)$$

$$= \frac{n}{2}(105 + 995)$$

$$= \frac{179}{2}(1100)$$

$$= \frac{196900}{2}$$

$$= 98450$$

$$\begin{array}{r} 179 \\ \times 11 \\ \hline 179 \\ 179 \\ \hline 196900 \end{array}$$

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Q.3 $a_1 = a = 2$ $a_{20} = ? = -112$

$\underbrace{a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5} \quad \underbrace{a_6 \quad a_7 \quad a_8 \quad a_9 \quad a_{10}}$

ATQ.

$$(a_1 + a_2 + a_3 + a_4 + a_5) = \frac{1}{4}(a_6 + a_7 + a_8 + a_9 + a_{10})$$

$$\Rightarrow \begin{pmatrix} a + a+d + a+2d \\ + a+3d + a+4d \end{pmatrix} = \frac{1}{4}\begin{pmatrix} a+5d + a+6d + a+7d \\ + a+8d + a+9d \end{pmatrix}$$

$$\Rightarrow \overset{a+2d}{\cancel{(5a+10d)}} = \frac{1}{4}\overset{a+7d}{\cancel{(5a+35d)}}$$

$$\Rightarrow 4a + 8d = a + 7d$$

$$\Rightarrow 3a + d = 0$$

$$\Rightarrow d = -3a$$

$a=2$

$$d = -3(2)$$

$$d = -6$$

$$a_{20} = a + 19 \cdot d$$

$$= 2 + 19(-6)$$

$$= 2 + (-114)$$

$$a_{20} = -112$$

Q.4

$-6, -\frac{11}{2}, -5, \ldots = \text{A.P.}$

$a = -6$

$\underline{\text{Sum} = -25}$

How many Terms $= ? = n$

Let the no. of terms $= n$

$S_n = \text{Sum} = -25$

$\Rightarrow \frac{n}{2}(2a + (n-1)\cdot d) = -25$

$\Rightarrow n\left[2\times(-6) + (n-1)\cdot\frac{1}{2}\right] = -25\times 2$

$\Rightarrow n\left(-12 + \frac{n-1}{2}\right) = -50$

$\Rightarrow n\left(\frac{-24+n-1}{2}\right) = -50$

$\Rightarrow n(n-25) = -100$

$d = a_2 - a_1$

$d = \left(-\frac{11}{2}\right) - (-6)$

$= -\frac{11}{2} + 6$

$= \frac{-11+12}{2} = \frac{1}{2}$

$n^2 - 25n + 100 = 0$

$\Rightarrow \underbrace{n^2 - 20n}\underbrace{-5n + 100} = 0$

$\Rightarrow n(n-20) - 5(n-20) = 0$

$\Rightarrow (n-20)(n-5) = 0$

$n = 20 \quad n = 5$

$\underbrace{a_1 \; a_2 \; a_3 \; a_4 \; a_5}_{-25} \; \underbrace{a_6 \; a_7 \ldots a_{20}}_{0}$

-25

n^{th} term $= a_n = a + (n-1) \cdot d$

$a_n = S_n - S_{n-1}$

$S_n = \frac{n}{2}(2a + (n-1) \cdot d)$ $a = a_1$ $d = C.D.$

$S_n = \frac{n}{2}(a + l) = \frac{n}{2}(a_1 + a_n)$

Q.6 $\boxed{25, 22, 19, \ldots, a_n}$ $a = a_1 = 25$

$S_n = 116$ $d = -3$

no. of terms $= n$

let last term $= a_n$

$\Rightarrow S_n = 116$

$\Rightarrow \frac{n}{2}(2a + (n-1) \cdot d) = 116$

$\Rightarrow n(2 \times 25 + (n-1)(-3)) = 232$

$\Rightarrow n \cdot (50 - 3n + 3) = 232$

$\Rightarrow n(53 - 3n) = 232$

$\Rightarrow -3n^2 + 53 \cdot n = 232$

$\Rightarrow 3n^2 - 53 \cdot n + 232 = 0$

$\Rightarrow$ $\boxed{-24} \times \boxed{-29}$

3×232
$= 3 \times 2 \times 116$
$= 3 \times 2 \times 2 \times 58$
$= \underbrace{3 \times 2 \times 2 \times 2}_{} \times 29$
$= 24 \times 29$

$\Rightarrow \underbrace{3n^2 - 24n} - 29n + 232 = 0$

$\Rightarrow 3n(n-8) - 29(n-8) = 0$

$\Rightarrow (n-8)(3n-29) = 0$

$n = 8 \in N$ ✓ $n = \frac{29}{3} \notin N$ ✗

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$a_n = a + (n-1).d$

Last term $a_8 = 25 + (8-1).(-3)$

$= 25 + 7.(-3)$

$= 25 - 21$

$\boxed{a_8 = 4}$

Q. 7 K^{th} term $= a_K = 5K+1$

AP $\rightarrow a_1, a_2, a_3, a_4, - - -$

$K=1$ $a_1 = 5+1 = 6 \longrightarrow a = 6$

$K=2$ $a_2 = 5\times2+1 = 11$ $d = 11-6 = 5$

$K=3$ $a_3 = 5\times3+1 = 16$

$\vdots$

$S_n = \frac{n}{2}(2a + (n-1).d) = \frac{n}{2}[2\times6 + (n-1)5]$

$S_n = \frac{n}{2}(12 + 5n - 5) = \frac{n}{2}(5n+7) = \frac{5n^2}{2} + \frac{7n}{2}$

Q. 8 $S_n = p.n + q.n^2$ p, q = Constants

Common Difference $= ? = d = a_2 - a_1$

$S_n = p.n + q.n^2 = \underline{a_1 + a_2 + \dots + a_n}$

$S_1 = \boxed{a_1 = p + q}$ — (1)

$S_2 = \boxed{a_1 + a_2 = 2p + 4.q}$ — (2)

By eq^n (1) & (2): eq^n(2) − eq^n(1)

$\boxed{a_2 = p + 3q}$

C.D. $= d = a_2 - a_1$

$d = (\cancel{p} + 3q) - (\cancel{p} + q)$

$\boxed{d = 2q}$

Note: $d = 2\times$ coeff. of n^2

S_n

Q.9 2 APs $\to a_1, a_2, a_3 \ldots \to$ First term $= a$, CD $= d$ (let) $\to$ Sum $= S_n^{I}$

$\to b_1, b_2, b_3 \ldots \to$ first term $= b$, CD $= e$ (let) $\to$ Sum $= S_n^{II}$

ratio of sum of n-terms $= (5n+4):(9n+6)$

ratio of their 18^{th} terms = ?

$$\to \frac{a_{18}}{b_{18}} = \frac{a+17.d}{b+17.e} = ?$$

Given, ratio of sum of n-terms $= \dfrac{5n+4}{9n+6}$

$$\Rightarrow \frac{S_n^{I}}{S_n^{II}} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\cancel{\frac{n}{2}}[2a+(n-1).d]/2}{\cancel{\frac{n}{2}}[2b+(n-1).e]/2} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{a+\left(\frac{n-1}{2}\right).d}{b+\left(\frac{n-1}{2}\right).e} = \frac{5n+4}{9n+6} \quad \text{--- (1)}$$

Let $\left(\frac{n-1}{2}\right) = 17$

$\Rightarrow n-1 = 34$

$\Rightarrow n = 35$

put $n=35$ in eq^n (1)

$$\Rightarrow \frac{a+17.d}{b+17.e} = \frac{5\times35+4}{9\times35+6}$$

$$\Rightarrow \frac{a_{18}}{b_{18}} = \frac{175+4}{315+6}$$

$$= \frac{179}{321}$$

$a_{18} : b_{18} = 179 : 321$

Q.10 $S_p = S_q$

$$S_n = \frac{n}{2}\left(2a + (n-1)\cdot d\right)$$

? let ? let

$$\Rightarrow \frac{p}{\cancel{2}}\left[2a + (p-1)\cdot d\right] = \frac{q}{\cancel{2}}\left[2a + (q-1)\cdot d\right]$$

$$\Rightarrow 2ap + (p-1)\cdot pd = 2aq + (q-1)\cdot qd$$

$$\Rightarrow 2a(p-q) + d\cdot\left[p^2 - p - q^2 + q\right] = 0$$

$$\Rightarrow 2a(p-q) + d\cdot\left[(p-q)\cdot(p+q) - (p-q)\right] = 0$$

$$\Rightarrow 2a\cdot(p-q) + d\cdot(p-q)\cdot\left[p+q-1\right] = 0$$

$$\Rightarrow (p-q)\cdot\left\{2a + d\cdot\left[p+q-1\right]\right\} = 0$$

$\cancel{0}$, $p \neq q$; $\{2a + d\cdot[p+q-1]\} = 0$

Sum of first $(p+q)$ terms $= S_{p+q} = \dfrac{p+q}{2}\left[\underbrace{2a + (p+q-1)\cdot d}_{0}\right]$

$$S_{p+q} = \left(\frac{p+q}{2}\right)(0)$$

$$= 0$$

Q.11 $\quad S_p = a \qquad S_q = b \qquad S_r = c$

First term $= A$
$CD = D$ } let

$\frac{q}{p}(q-r)$

(A, D)

$S_p = a$

$\Rightarrow \frac{p}{2}\left(2A + (p-1)\cdot D\right) = a$

$\Rightarrow \frac{1}{2}\left(2A + (p-1)\cdot D\right) = \frac{a}{p}$

$\Rightarrow \frac{a}{p} = A + \frac{p\cdot D}{2} - \frac{D}{2}$

$\Rightarrow \boxed{\frac{a}{p} = A - \frac{D}{2} + \frac{p\cdot D}{2}}$

$(q-r)$ multiply

$\Rightarrow \frac{a}{p}(q-r) = \left(A - \frac{D}{2} + \frac{pD}{2}\right)\cdot(q-r)$

$\Rightarrow \frac{a}{p}(q-r) = A\cdot q - Ar - \frac{D}{2}\cdot q + \frac{D}{2}\cdot r + \frac{D}{2}\cdot pq - \frac{D}{2}\cdot pr$

$\frac{a}{p}(q-r) = \cancel{Aq} - \cancel{Ar} - \cancel{\frac{D}{2}q} + \cancel{\frac{D}{2}\cdot r} + \cancel{\frac{D}{2}\cdot pq} - \cancel{\frac{D}{2}\cdot pr}$

$\frac{b}{q}(r-p) = \cancel{A\cdot r} - \cancel{A\cdot p} - \cancel{\frac{D}{2}\cdot r} + \cancel{\frac{D}{2}\cdot p} + \cancel{\frac{D}{2}\cdot qr} - \cancel{\frac{D}{2}\cdot pq}$

$\frac{c}{r}(p-q) = \cancel{A\cdot p} - \cancel{A\cdot q} - \cancel{\frac{D}{2}\cdot p} + \cancel{\frac{D}{2}\cdot q} + \cancel{\frac{D}{2}\cdot pr} - \cancel{\frac{D}{2}\cdot qr}$

$+$

$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

H.P.

Revision
$$\begin{cases} a_n = a+(n-1)\cdot d \\ a_n = S_n - S_{n-1} \\ S_n = \frac{n}{2}(2a+(n-1)\cdot d) \\ S_n = \frac{n}{2}(a+l) \end{cases}$$

$\boxed{Q. 12}$ an A.P. → first term = a ; C.D. = d (Let)

Ratio of sums of m & n terms = $m^2 : n^2$

To prove → $\boxed{a_m : a_n = (2m-1) : (2n-1)}$

$$\frac{S_m}{S_n} = \frac{m^2}{n^2}$$

$$\frac{a_m}{a_n} = \frac{a+(m-1)\cdot d}{a+(n-1)\cdot d}$$

$$\Rightarrow \frac{\cancel{\frac{m}{2}}(2a+(m-1)\cdot d)}{\cancel{\frac{n}{2}}(2a+(n-1)\cdot d)} = \frac{m^{\cancel{2}}}{n^{\cancel{2}}}$$

$$\Rightarrow \frac{[2a+(m-1)\cdot d]/2}{[2a+(n-1)\cdot d]/2} = \frac{m}{n}$$

$$\Rightarrow \frac{a+\left(\frac{m-1}{2}\right)\cdot d}{a+\left(\frac{n-1}{2}\right)\cdot d} = \frac{m}{n} \quad \text{—} (1)$$

Let $\frac{m-1}{2} \to (M-1)$ substitute.

$\frac{n-1}{2} \to (N-1)$ substitute.

$$\frac{m-1}{2} = M-1$$

$$m = 2M-2 \;+1$$

$$m = 2M-1$$

$$n = 2N-1$$

By substitution in eq^n (1):

$$\Rightarrow \frac{a+(M-1).d}{a+(N-1).d} = \frac{2M-1}{2N-1}$$

$$\Rightarrow \frac{a_M}{a_N} = \frac{2M-1}{2N-1}$$

$M \to m$
$N \to n$

$$\Rightarrow \frac{a_m}{a_n} = \frac{2m-1}{2n-1}$$ H.p.

(13) $S_n = 3n^2 + 5n$

$a_m = m^{th}$ term $= 164$, $m = ?$

$S_n = 3n^2 + 5n = a_1 + a_2 + \dots + a_n$

$S_1 = a_1 = 3 + 5 = 8 =$ First term.

$S_2 = a_1 + a_2 = 3 \times 4 + 5 \times 2 = 12 + 10 = 22$

$\cancel{a_1+8}$ $a_1 + a_2 = 22$

$8 + a_2 = 22 \Rightarrow \boxed{a_2 = 14}$

$a_1 = 8 = a$
$a_2 = 14$
$d = a_2 - a_1 = 14 - 8 = 6$

m^{th} term $= 164$

$\Rightarrow a_m = a + (m-1).d = 164$

$\Rightarrow 8 + (m-1).6 = 164$

$\Rightarrow (m-1).6 = 156$

$\Rightarrow m - 1 = \frac{\cancel{156}^{26}}{\cancel{6}}$

$\Rightarrow \boxed{m = 27}$

Revision:

$\boxed{a}$, A_1, A_2 . . . A_n, $\boxed{b}$ → AP

A.M.

First Term: a

$a+d$, $a+2d$, $a+n.d$

Last Term: $a+(n+1).d$

unknown $= d \Rightarrow$?

$b = a+(n+1).d$?

$\boxed{Q.14}$

$8 \xrightarrow{d} \boxed{A_1} \xrightarrow{d} \boxed{A_2}\ \boxed{A_3}\ \boxed{A_4}\ \boxed{A_5}\ 26 \longrightarrow$ A.P.

C.D $= d$ let

8, $8+d$, $8+2d$, $8+3d$, $8+4d$, $8+5d$, $8+6d$

11, 14, 17, 20, 23

$26 = 8+6d$

$\Rightarrow 18 = 6d$

$\Rightarrow \boxed{3 = d}$

$5 \quad 9$

$\boxed{Q.15}$ $a \neq b$

$\underbrace{\text{A.M. between } a \ \& \ b}_{\text{Self}} = \dfrac{a^n + b^n}{a^{n-1}+b^{n-1}}$

$$\Rightarrow \frac{a+b}{2} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$

$$\Rightarrow \cancel{a^n} + a.b^{n-1} + b.a^{n-1} + \cancel{b^n} = \cancel{2}a^n + \cancel{2}b^n$$

$$\Rightarrow a.b^{n-1} + a^{n-1}.b = a^n + b^n$$

$$\Rightarrow a^n - a^{n-1}.b + b^n - a.b^{n-1} = 0$$

$$\Rightarrow a^{n-1}.\{a-b\} - b^{n-1}\{a-b\} = 0$$

$$\Rightarrow (a-b).\{a^{n-1} - b^{n-1}\} = 0$$

$a - b \neq 0$; $a^{n-1} - b^{n-1} = 0$

$a \neq b$ | $\boxed{a^{n-1} = b^{n-1}}$

$n-1 = 0$

$n = 1$

Q.16

$1, \underset{1+d}{\boxed{A_1}}, \underset{1+2d}{\boxed{A_2}}, \text{------}, \boxed{A_{m-1}}, \underset{1+m.d}{\boxed{A_m}}, \underset{1+(m+1).d}{31} \longrightarrow$ A.P.

CD = d

$A_7 : A_{m-1} = 5 : 9 \longrightarrow m = ?$

$31 = 1 + (m+1).d$

$\Rightarrow 30 = (m+1).d$

$\Rightarrow \frac{30}{(m+1)} = d$

$A_7 = 1 + 7d = 1 + 7 \times \frac{30}{m+1} = 1 + \frac{210}{m+1}$

$A_{m-1} = 1 + (m-1).d = 1 + \frac{30}{m+1}(m-1)$

$\frac{A_7}{A_{m-1}} = \frac{5}{9}$

$\Rightarrow \frac{1 + \frac{210}{m+1}}{1 + \frac{30(m-1)}{m+1}} = \frac{5}{9}$

$\Rightarrow \frac{\frac{m+1+210}{\cancel{m+1}}}{\frac{m+1+30(m-1)}{\cancel{m+1}}} = \frac{5}{9}$

$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$

$\Rightarrow 9m + 1899 = 155m - 145$

$\Rightarrow 2044 = 146.m$

$\Rightarrow m = \frac{2044}{146} = 14$

$\begin{array}{r} 1900 \\ 144 \\ \hline 2044 \end{array}$

Q.17

1^{st} instalment $\rightarrow$ 100

2^{nd} instalment $\rightarrow$ 105 $\quad$ +5

⋮

30^{th} instalment $\rightarrow$ a_{30}

$a = 100$

$d = 5$

$a_{30} = a + 29.d$

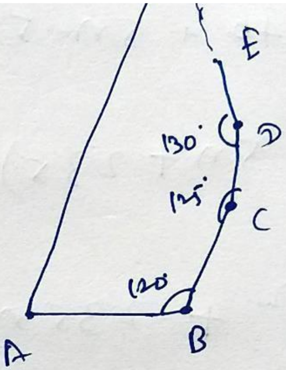
$a_{30} = 100 + 29 \times 5$

$a_{30} = 100 + 145$

$a_{30} = 245$

Q.18

Polygon



Angle: $120°, 125°, 130°, 135° \ldots a_n$

n-sided polygon $\rightarrow$ angles $= a_n$

$$\text{Sum of interior angles of n-sided polygon} = (n-2).180°$$

$$\underbrace{(120° + 125° + 130° + \ldots + a_n)}_{S_n} = \text{Sum of interior Angles} = (n-2).180°$$

$$\Rightarrow \frac{n}{2}\left[2 \times 120 + (n-1).5\right] = (n-2).180°$$

$$\Rightarrow n(240 + 5n - 5) = 360n - 720°$$

$\Rightarrow n(240 + 5n - 5) = 360n - 720$

$\Rightarrow n(5n + 235) = 360n - 720$

$\Rightarrow 5n^2 + 235n = 360n - 720$

$\Rightarrow 5n^2 + \underline{235n - 360n} + 720 = 0$

$\Rightarrow \dfrac{5n^2 - 125n + 720 = 0}{5 \qquad\qquad 5}$

$\Rightarrow \underline{n^2 - 25n + 144 = 0}$

$\Rightarrow \underbrace{n^2 - 9n}\; - \underbrace{16n + 144} = 0$

$\Rightarrow \cancel{n(+9} \; n(n-9) - (16)(n-9) = 0$

$(n-9)(n-16) = 0$

$n = 9$, ~~$n = 16$~~ ✗

$\underline{n = 9}$,

Last Angle $= a_9$

$= a + 8d$

$= 120 + 8 \times 5$

$= 120 + 40$

$= 160°$

$n = 9$ ✓

$\underline{n = 16}$

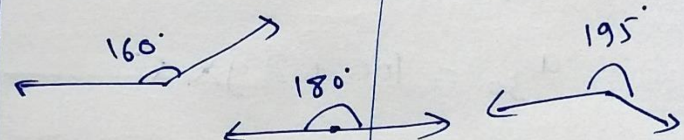
Last angle $= a_{16}$

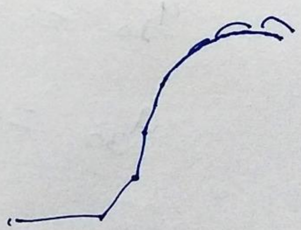
$= a + 15 \cdot d$

$= 120 + 15 \times 5$

$= 120 + 75$

$= 195°$





✗

not possible

Geometric Progression
(G.P.)

ratio of two consecutive terms = Constant (r)

$$\frac{a_{n+1}}{a_n} = r \Rightarrow \boxed{a_{n+1} = r \cdot a_n}, \; r \neq 0$$

General Term:
(n^{th} term $= a_n$)

First term $= a$
Common ratio $= r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$

$$\begin{array}{cccccc} a, & ar, & ar^2, & ar^3 & \ldots & (a.r^{n-1}) \\ \downarrow & \downarrow & \downarrow & \downarrow & & \uparrow \\ a_1 & a_2 & a_3 & a_4 & & a_n \end{array}$$

General Term

$\boxed{a_n = a \cdot r^{n-1}} \leftarrow$ for G.P.

$a_n = a + (n-1) \cdot d \leftarrow$ for A.P.

Sum of n-terms of G.P. $= \boxed{S_n = \frac{a \cdot (r^n - 1)}{(r-1)} = \frac{a(1-r^n)}{(1-r)}}$

$$S_n = a + \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} \ldots\ldots + \cancel{a.r^{n-1}} \quad \text{---} (1)$$

$$r.S_n = \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \ldots\ldots + \cancel{a.r^{n-1}} + a.r^n$$

$$S_n(1-r) = a - a.r^n = a(1-r^n) \Longrightarrow S_n.(1-r) = a.(1-r^n)$$

?

Properties of G.P.

① $a_n = a.r^{n-1}$

② $S_n = \dfrac{a.(r^n - 1)}{(r-1)}$

③ If a, b, c are in G.P. ⇒ $\boxed{b^2 = a.c}$ or $\boxed{b = \sqrt{ac}}$

④ (in each term)

G.P. → $+K$ → } we do not get a G.P. again.

G.P. → $-K$ →

G.P. → $\times K$ → } we get a new G.P.

G.P. → $\div K$ →

⑤ Assumption. (we have the product of numbers)

3 No. in G.P. → $\frac{a}{r}, a, ar$ 4 no. in G.P. → $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$$\ldots \frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3, \ldots$$

Geometric mean (GM)

(i) G.M. of $a, b = \sqrt{a.b}$

$= (ab)^{\frac{1}{2}}$

GM of $a, b, c = (a.b.c)^{\frac{1}{3}} = \sqrt[3]{a.b.c}$

GM of $a, b, c, d = (a.b.c.d)^{\frac{1}{4}}$

(ii) n-GMs → Find → between a & b

Let $G_1, G_2, G_3, \ldots G_n$ are the n- Geometric means between a & b

Such that $a, G_1, G_2, G_3, \ldots, G_n, b$ form a G.P.

$a = a \quad a.r \quad a.r^2 \quad ar^3 \quad a.r^n \quad a.r^{n+1}$

unknown $= r$

$$b = a.r^{n+1}$$

$$\frac{b}{a} = r^{n+1}$$

$$\left(\frac{b}{a}\right)^{\frac{1}{n+1}} = r$$

Relation between AM & GM of two positive* Real numbers (a & b) : →

$AM = \frac{a+b}{2}$, $GM = \sqrt{ab}$

* $\boxed{AM \geq GM}$

* $\frac{a+b}{2} \geq \sqrt{ab}$

e.g. $a=2, b=4$

$\frac{2+4}{2} \geq \sqrt{2\times4}$

$\frac{6}{2} \geq \sqrt{8}$

$3 \geq 2\sqrt{2}$

2×1.414

$3 \geq 2.828\ldots$

e.g. $a=b=2$

$\frac{2+2}{2} \geq \sqrt{2\times2}$

$2 \geq 2$

$2=2$ ✓

Proof.

Square of any Real No. ≥ 0

$\Rightarrow (\sqrt{a}-\sqrt{b})^2 \geq 0$

$\Rightarrow (\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{a}\sqrt{b} \geq 0$

$\Rightarrow a + b - 2\sqrt{ab} \geq 0$

$\Rightarrow a+b \geq 2\sqrt{ab}$

$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$

$AM \geq GM$

$a, b > 0$

Exercise 8.3

Revision:

G.P.

First term $= a$

C.R. $= r$

① $a_n = a.r^{n-1}$

② $S_n = \dfrac{a.(r^n - 1)}{(r-1)}$

③ $a, b, c \to$ G.P.

$\therefore \boxed{b^2 = a.c}$

Q.1 G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

First term $= a = \frac{5}{2}$

C.R. $= r = \dfrac{a_2}{a_1} = \dfrac{\cancel{5}/4}{\cancel{5}/2} = \dfrac{2}{4} = \dfrac{1}{2}$

20^{th} term $= a_{20} = a.r^{20-1} = \frac{5}{2} \times \left(\frac{1}{2}\right)^{19}$

$= \frac{5}{2} \times \frac{1}{2^{19}} = \frac{5}{2^{20}}$

n^{th} term $= a_n = a.r^{n-1}$

$= \frac{5}{2} \cdot \left(\frac{1}{2}\right)^{n-1} = \dfrac{5}{2^1 \times 2^{n-1}} = \dfrac{5}{2^n}$

Q.2 $a_8 = 192$

$r = 2$

$a_{12} = ?$

$a_{12} = a.r^{12-1}$

$= a.r^{11}$

$a_8 = 192$

$\Rightarrow a.r^{8-1} = 192$

$\Rightarrow a.2^7 = 192$

$\Rightarrow a = \dfrac{192}{2^7} = \dfrac{3 \times \cancel{64}}{2 \times \cancel{64}} = \dfrac{3}{2}$

$a_{12} = a.r^{11}$

$= \frac{3}{2} \times 2^{11}$

$= 3 \times 2^{10} = 3 \times 1024$

$= 3072$

$\boxed{Q.3}$ $a_5 = p = a.r^4$

$a_8 = q = a.r^7$ Prove $\boxed{q^2 = ps}$

$a_{11} = s = a.r^{10}$

Let first term = a
CR = r

$(a^m)^n = a^{m.n}$

$\cancel{LHS = q^2}$: $q^2 = p.s.$

$\Rightarrow (a.r^7)^2 = (a.r^4).(a.r^{10})$

$\Rightarrow a^2.(r^7)^2 = a.a.r^4.r^{10}$

$\Rightarrow \boxed{a^2.r^{14} = a^2.r^{14}}$

LHS = RHS.

Q.4 $a_4 = (a_2)^2$

$a_7 = ?$

$a.r^6$

First term = $\boxed{-3 = a}$

$a_4 = (a_2)^2$

$\Rightarrow a.r^3 = (a.r)^2$

$\Rightarrow \cancel{a}.\cancel{r^3}r = \cancel{a^2}a.\cancel{r^2}$

$\Rightarrow$

$r = a$

$\Rightarrow \boxed{r = -3}$

$a_7 = a.r^6$

$= (-3)^1.(-3)^6$

$= (-3)^7$

$= -.3^7$

$= -2187$

Rough,

$\cancel{3^4 = 81}$

$\cancel{3^3 = 27}$

$3^7 = 3^5 \times 3^2$

$= 243 \times 9$

$= 2187$

Q.5

(a) $2, 2\sqrt{2}, 4, \ldots\ldots, 128$

$a = 2$

$r = \frac{a_2}{a_1} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$

Let $a_n = 128$

13th term

$a_n = 128$

$\Rightarrow a \cdot r^{n-1} = 128$

$\Rightarrow 2 \cdot (\sqrt{2})^{n-1} = 2^7$

$\Rightarrow 2^1 \cdot (2^{1/2})^{n-1} = 2^7$

$\Rightarrow 2^1 \cdot 2^{\frac{n-1}{2}} = 2^7$

$\Rightarrow 2^{1+\frac{n-1}{2}} = 2^7$

$1 + \frac{n-1}{2} = 7$

$\Rightarrow \frac{n-1}{2} = 6$

$\Rightarrow n - 1 = 12$

$n = 13$

(b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots\ldots, 729$

a_n

$a = \sqrt{3}$

$r = \frac{3}{\sqrt{3}} = \frac{\sqrt{3} \cdot \cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = \sqrt{3}$

Let $a_n = 729$

12th term

$\Rightarrow a \cdot r^{n-1} = 729$

$\Rightarrow (\sqrt{3})^1 \cdot (\sqrt{3})^{n-1} = 729$

$\Rightarrow (\sqrt{3})^{\cancel{1}+n-\cancel{1}} = 3^6$

$\Rightarrow (\sqrt{3})^n = 3^6$

$\Rightarrow (3)^{n/2} = 3^6$

$\frac{n}{2} = 6$

$n = 12$

Q.5 (c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{19683}$

9^{th} term

$a = \frac{1}{3}$

$r = \frac{\left(\frac{1}{9}\right)}{\left(\frac{1}{3}\right)} = \frac{3}{9} = \frac{1}{3}$

Let $a_n = \frac{1}{19683}$

$\Rightarrow a \cdot r^{n-1} = \frac{1}{19683}$

$\Rightarrow \left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$

$\Rightarrow \frac{1}{3^1 \times 3^{n-1}} = \frac{1}{19683}$

$\Rightarrow \frac{1}{3^{n}} = \frac{1}{3^{9}}$

$n = 9$

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$
$3^7 = 2187$ $\times 9$
$3^8 =$
$3^9 = 19683$

Q.6 $-\frac{2}{7}, x, -\frac{7}{2} \longrightarrow$ G.P.

$a, b, c \longrightarrow$ GP

$b^2 = a \times c$

$x^2 = -\frac{\cancel{2}}{\cancel{7}} \times -\frac{\cancel{7}}{\cancel{2}}$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm\sqrt{1}$

$x = \pm 1$

G.P. $a_n = a \cdot r^{n-1}$ $CR = r$

$a_1 = a$

$S_n = \frac{a \cdot (r^n - 1)}{(r-1)} = \frac{a(1-r^n)}{(1-r)}$

Q.7 $\underline{0.15}, \underline{0.015}, 0.0015 \ldots\ldots 20$ terms

$a = 0.15$

$r = \frac{0.015}{0.15} = 0.1 = \frac{\cancel{15}}{\cancel{15}} \times \frac{\cancel{100}}{\cancel{1000}}$

$n = 20$

$S_{20} = \frac{a(1-r^{20})}{1-r}$

$= \frac{0.15 \cdot [1-(0.1)^{20}]}{1-0.1}$

$= \frac{\overset{0.05}{\cancel{0.15}} \times [1-(0.1)^{20}]}{\cancel{0.9}\ 0.3}$

$= \frac{0.5(1-0.1^{20})}{3}$

Q.8 $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots$ n terms

$a = \sqrt{7}$

$r = \frac{\sqrt{21}}{\sqrt{7}} = \frac{\sqrt{\cancel{7} \times 3}}{\sqrt{\cancel{7}}} = \sqrt{3}$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_n = \frac{\sqrt{7}((\sqrt{3})^n - 1)}{\sqrt{3} - 1}$ $\sqrt{3} = 3^{\frac{1}{2}}$

$$\boxed{S_n = \frac{\sqrt{7}(3^{n/2} - 1)}{\sqrt{3} - 1}}$$

Q.9 $1, -a, a^2, -a^3, \dots$ n terms.

$a = 1$

$r = \frac{-a}{1} = -a$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_n = \frac{1.((-a)^n - 1)}{-a-1}$

$S_n = \frac{1-(-a)^n}{a+1}$

Q.10

$x^3, x^5, x^7, \dots$ n terms

$a = x^3$

$r = \frac{x^5}{x^3} = x^2$

$S_n = \frac{a.(r^n - 1)}{r-1}$

$S_n = \frac{x^3[(x^2)^n - 1]}{x^2-1} = \frac{x^3(x^{2n} - 1)}{x^2-1}$

Q.11

$$\sum_{K=1}^{11} (2+3^K)$$

$$= \underset{K=1}{(2+3^1)} + \underset{K=2}{(2+3^2)} + \underset{K=3}{(2+3^3)} + \cdots + \underset{K=11}{(2+3^{11})}$$

$$= (\underbrace{2+2+2+\cdots+2}_{11\text{-terms.}}) + (\underbrace{3^1+3^2+3^3+\cdots+3^{11}}_{11\text{- terms}})$$

G.P. $\longrightarrow S_n = \dfrac{a.(r^n-1)}{r-1}$

$a=3$
$r=3$
$n=11$

$$= 22 + \frac{3.(3^{11}-1)}{(3-1)}$$

$$= 22 + \frac{3}{2}.(3^{11}-1)$$

Q. 12

$a_1 + a_2 + a_3 = S_3 = \frac{39}{10}$

Their product $= 1$

$r = ?$

Terms $= ?$

G.P.

$\frac{a}{r^2}, \boxed{\frac{a}{r}, a, ar}\ ar^2, ar^3, \ldots$

Let

$a_1 = \frac{a}{r} = \frac{1}{r}$

$a_2 = a = 1$

$a_3 = ar = r$

Common ratio $= r$

Given ~~tep~~ their product $= 1$

$\Rightarrow a_1 \times a_2 \times a_3 = 1$

$\Rightarrow \frac{a}{\cancel{r}} \cdot a \cdot a\cancel{r} = 1$

$\Rightarrow a^3 = 1$

$\Rightarrow \boxed{a = 1}$

Sum $= a_1 + a_2 + a_3 = \frac{39}{10}$

$\Rightarrow \frac{1}{r} + 1 + r = \frac{39}{10}$

$\Rightarrow \frac{1 + r + r^2}{r} = \frac{39}{10}$

$\Rightarrow 10 + 10.r + 10.r^2 = 39.r$

$\Rightarrow 10.r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$

$\Rightarrow (5r - 2).(2r - 5) = 0$

$r = \frac{2}{5}, \frac{5}{2}$

Numbers

$a_1 = \frac{1}{r} = \frac{1}{(2/5)} = \frac{5}{2}$

$a_2 = 1 = 1$

$a_3 = r = \frac{2}{5}$

G.P.

$a_n = a \cdot r^{n-1}$

$S_n = \frac{a \cdot (r^n - 1)}{(r-1)}$

Q. 13

$3, 3^2, 3^3, \ldots, a_n$ $\quad n = ?$

Sum $= 120 = S_n$

$a = 3$
$r = 3$

$S_n = \frac{\cancel{3} \cdot (3^n - 1)}{3-1} = \cancel{120}\ 40$

$\Rightarrow \frac{3^n - 1}{2} = 40$

$\Rightarrow 3^n - 1 = 80$

$\Rightarrow 3^n = 81 = 3^4$

$\Rightarrow \boxed{n = 4}$

Q. 14

$\underbrace{a, ar, ar^2}_{16}, \underbrace{ar^3, ar^4, ar^5}_{128}, ar^6, \ldots$

$a + ar + ar^2 = 16 \Rightarrow a(1 + r + r^2) = 16$ — (1)

$ar^3 + ar^4 + ar^5 = 128 \Rightarrow ar^3(1 + r + r^2) = 128$ — (2)

$\frac{\text{eq}^n (1)}{\text{eq}^n (2)} :\rightarrow \frac{\cancel{a}\cancel{(1+r+r^2)}}{\cancel{a}r^3\cancel{(1+r+r^2)}} = \frac{\cancel{16}}{\cancel{128}\ 8}$

$\Rightarrow \frac{1}{r^3} = \frac{1}{8} = \frac{1}{2^3}$

$\Rightarrow \boxed{r = 2}$

$\longrightarrow$ By eq^n (1) $\quad a(1 + 2 + 4) = 16$ $\quad \boxed{a = \frac{16}{7}}$

$S_n = \frac{a(r^n - 1)}{r-1} = \frac{\frac{16}{7} \times (2^n - 1)}{2-1} = \frac{16}{7} \times (2^n - 1)$

Q.15 G.P. $a = 729$, $a_7 = 64$, $S_7 = ?$

$a_7 = 64$

$\Rightarrow a \cdot r^6 = 64$

$\Rightarrow 729 \cdot r^6 = 64$

$\Rightarrow r^6 = \frac{64}{729} = \frac{2^6}{3^6} = \left(\frac{2}{3}\right)^6$

$\Rightarrow \boxed{r = \frac{2}{3}}$

$S_7 = \frac{a \cdot (r^7 - 1)}{r - 1}$

$= \frac{729 \cdot \left(\left(\frac{2}{3}\right)^7 - 1\right)}{\left(\frac{2}{3} - 1\right)} = \frac{729 \cdot \left(\left(\frac{2}{3}\right)^7 - 1\right)}{-\frac{1}{3}}$

$= 2187 \left[1 - \left(\frac{2}{3}\right)^7\right]$

Q.16 $\underbrace{a, ar}, ar^2, ar^3, ar^4, ar^5, \ldots$

$ar^2 \leftarrow a_3$, $ar^4 \leftarrow a_5$

$a + ar = -4$

$a_5 = 4 \cdot (a_3) \Rightarrow \cancel{a}r^{\cancel{4}} = 4 \cdot \cancel{a}\cancel{r^2}$

$r^2 = 4$

$\Rightarrow \boxed{r = \pm 2}$

$a + ar = -4$

$\Rightarrow a(1 + r) = -4$

$\Rightarrow a = \frac{-4}{1 + r}$

$r = 2$

$a = \frac{-4}{1+2} = \frac{-4}{3}$

G.P.

$\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots$

$r = -2$

$a = \frac{-4}{1 + (-2)} = \frac{-4}{-1} = 4$

$4, -8, 16, \ldots$

Q.17

G.P. $a_4, \; a_{10}, \; a_{16}$

$a \cdot r^3 \to x, \quad a \cdot r^9 \to y, \quad a \cdot r^{15} \to z$.

To Prove: x, y, z are also in G.P.

$y^2 = x \cdot z$

LHS $= y^2 = (a \cdot r^9)^2 = a^2 \cdot (r^9)^2 = a^2 \cdot r^{18}$

RHS $= x \cdot z = (a r^3) \cdot (a r^{15}) = a^2 \cdot r^{18}$

$\therefore \boxed{y^2 = xz}$

$\therefore x, y, z$ are in G.P.

Q.18 Sum of n-terms of

$8, 88, 888, 8888, \ldots$

$S_n = 8 + 88 + 888 + \cdots + $

$= 8(1 + 11 + 111 + \cdots)$

$= \frac{8}{9}(9 + 99 + 999 + \cdots)$

$= \frac{8}{9}\left[(10-1) + (100-1) + (1000-1) + \cdots\right]$

$= \frac{8}{9}\left[\overbrace{(10 + 100 + 1000 + \cdots)}^{\text{G.P.}} - (1 + 1 + 1 + \cdots)\right]$

$= \frac{8}{9}\left[\underbrace{[10^1 + 10^2 + 10^3 + \cdots + 10^n]}_{\text{G.P.}} - \underbrace{(1 + 1 + \cdots + 1)}_{\text{n-times}}\right]$

$= \frac{8}{9}\left[\frac{10 \cdot (10^n - 1)}{10 - 1} - n\right]$

$= \frac{8}{9}\left[\frac{10}{9} \cdot (10^n - 1) - n\right]$

$= \boxed{\frac{80}{81} \cdot (10^n - 1) - \frac{8n}{9}}$

G.P. $a_n = a \cdot r^{n-1}$

$S_n = \frac{a \cdot (r^n - 1)}{(r-1)}$

$\boxed{\text{Q. 19}}$

$G.P._{I} = 2, 4, 8, 16, 32$

$G.P._{II} = 128, 32, 8, 2, \frac{1}{2}$

ATQ. GP. $\rightarrow 256, 128, 64, 32, 16$

a_1, $r = \frac{1}{2}$

$\text{Sum} = S_5$

$= 256 \cdot \frac{\left(\left(\frac{1}{2}\right)^5 - 1\right)}{\left(\frac{1}{2} - 1\right)} = 256 \cdot \frac{\left(\frac{1}{32} - 1\right)}{-\frac{1}{2}}$

$= 256 \cdot \frac{\left(\frac{-31}{\cancel{32}_{16}}\right)}{\left(-\frac{1}{\cancel{2}}\right)} = \frac{\cancel{256}^{16} \times 31}{\cancel{16}} = 496$

$\boxed{\text{Q. 20}}$

$GP_{I} \rightarrow a, ar, ar^2, \ldots, ar^{n-1}$

$GP_{II} \rightarrow A, AR, AR^2, \ldots, AR^{n-1}$

Sequence. $aA, aArR, aA(rR)^2, aA(rR)^3, \ldots, aA(rR)^{n-1}$

G.P.

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots = \text{Common Ratio}$

$\frac{\cancel{aA}rR}{\cancel{aA}} = \frac{\cancel{aA}(rR)^2}{\cancel{aA}(rR)} = \ldots = CR$

$\Rightarrow rR = rR = \ldots = CR$

Common Ratio = rR

(21) $a_1, a_2, a_3, a_4 \longrightarrow$ G.P.

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$a, ar, ar^2, ar^3 \leftarrow$ Let.

$a_3 = a_1 + 9 \Rightarrow ar^2 = a + 9 \Rightarrow ar^2 - a = 9 \Rightarrow a(r^2-1) = 9$ — (1)

$a_2 = a_4 + 18 \Rightarrow ar = ar^3 + 18 \Rightarrow ar - ar^3 = 18 \Rightarrow -ar(r^2-1) = 18$ — (2)

$\dfrac{\text{eq}^n (1)}{\text{eq}^n (2)} :\rightarrow \dfrac{\cancel{a}\cancel{(r^2-1)}}{-\cancel{a}r.\cancel{(r^2-1)}} = \dfrac{\cancel{9}}{\cancel{18}_2}$

$\Rightarrow -\dfrac{1}{r} = \dfrac{1}{2} \Rightarrow \boxed{r = -2} \longrightarrow$ By eq^n (1).

$a((-2)^2-1) = 9$

$\Rightarrow a(\cancel{3}) = \cancel{9}_3$

$\Rightarrow \boxed{a = 3}$

Four No. $a = 3, r = -2$

$a_1 = a = 3$

$a_2 = ar = -6$

$a_3 = ar^2 = 12$

$a_4 = ar^3 = -24$

$\boxed{Q.22.}$

GP. let

$a_1 = A$
$CR = R$

p^{th} term $\longrightarrow a = A.R^{p-1}$

q^{th} term $\longrightarrow b = A.R^{q-1}$

r^{th} term $\longrightarrow c = A.R^{r-1}$

$\boxed{\text{To Prove}} \rightarrow a^{q-r} . b^{r-p} . c^{p-q} = 1$

$$LHS = a^{q-r} . b^{r-p} . c^{p-q}$$

$$= \left(A.R^{p-1}\right)^{q-r} . \left(A.R^{q-1}\right)^{r-p} . \left(A.R^{r-1}\right)^{p-q}$$

$$= A^{(\cancel{q}-\cancel{r})+(\cancel{r}-\cancel{p})+(\cancel{p}-\cancel{q})} \times R^{\cancel{pq}-\cancel{pr}-\cancel{q}+\cancel{r}+\cancel{qr}-\cancel{pq}-\cancel{r}+\cancel{p}+\cancel{pr}-\cancel{qr}-\cancel{p}+\cancel{q}}$$

$$= A^0 \times R^0$$

$$= 1 \times 1 = 1 = RHS.$$

Q. 23

$a_1 = a$

$a_n = b$

Prove $\boxed{P^2 = (ab)^n}$

P = product of n terms.

Let common Ratio = r

$a_1 = a$

$a_n = b = a \cdot r^{n-1}$

$P = a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdots a_n$

$P = (a) \cdot (ar) \cdot (ar^2) \cdot (ar^3) \cdots (a \cdot r^{n-1})$

$= \underbrace{(a \cdot a \cdot a \cdots a)}_{n\text{-times}} \cdot (\underbrace{r^1 \cdot r^2 \cdot r^3 \cdots r^{n-1}})$

$= (a^n) \cdot \left(r^{\overbrace{1+2+3+\cdots+(n-1)}^{A.P.}} \right)$

$S_{n-1} = \frac{n-1}{2}\left(2 \times 1 + (n-2) \cdot 1\right)$

A.P.

$= \frac{n-1}{2}(\cancel{2} + n - \cancel{2})$

$= \frac{(n-1) \cdot n}{2}$

$P = a^n \cdot r^{\frac{(n-1) \cdot n}{2}}$

$\text{LHS} = P^2 = \left[a^n \cdot r^{\frac{n(n-1)}{2}}\right]^2$

$= a^{2n} \cdot r^{n(n-1)}$ — (1)

$\text{RHS} = (ab)^n$

$= (a \cdot a \cdot r^{n-1})^n$

$= [a^2 \cdot r^{n-1}]^n$

$= a^{2n} \cdot r^{n(n-1)}$ — (2)

LHS = RHS

$P^2 = (ab)^n$

Q.24

$\boxed{a}$ $\boxed{ar}$ $\boxed{ar^2}$ $\boxed{a.r^{n-1}}$ $\boxed{a.r^n}$ $\boxed{a.r^{n+1}}$ $\boxed{a.r^{2n-1}}$

$$\underbrace{a_1, a_2, a_3, \dots a_n,}_{Sum_1} \underbrace{a_{n+1}, a_{n+2}, \dots a_{2n},}_{Sum_2}$$

To Prove $\boxed{\frac{Sum_1}{Sum_2} = \frac{1}{r^n}}$

$$Sum_1 = S_n^{I} = \frac{a.(r^n-1)}{r-1}$$

$$Sum_2 = S_n^{II} = \frac{a.r^n.(r^n-1)}{r-1}$$

$$\frac{Sum_1}{Sum_2} = \frac{\frac{\cancel{a}\cancel{(r^n-1)}}{\cancel{(r-1)}}}{\frac{\cancel{a}.r^n.\cancel{(r^n-1)}}{\cancel{(r-1)}}} = \frac{1}{r^n}$$

Q.25

$a, b, c, d \longrightarrow$ G.P.

let a, ar, ar^2, ar^3 } Put

To Prove:

$$(a^2+b^2+c^2).(b^2+c^2+d^2) = (ab+bc+cd)^2$$

$$LHS = (a^2+b^2+c^2).(b^2+c^2+d^2)$$
$$= (a^2+a^2r^2+a^2r^4).(a^2r^2+a^2r^4+a^2r^6)$$
$$= a^2(1+r^2+r^4).a^2.r^2(1+r^2+r^4)$$
$$= a^4.r^2.(1+r^2+r^4)^2$$

$$RHS = (ab+bc+cd)^2$$
$$= [a.ar + ar.ar^2 + ar^2.ar^3]^2$$
$$= [a^2r + a^2r^3 + a^2r^5]^2$$
$$= [r.a^2.\{1+r^2+r^4\}]^2$$
$$= a^4r^2.(1+r^2+r^4)^2$$

LHS = RHS

$\boxed{Q.26}$

$3, \boxed{G_1}, \boxed{G_2}, 81 \rightarrow$ G.P.

$G_1 \downarrow 3.r$, $G_2 \downarrow 3r^2$, $81 \downarrow 3r^3$

Let $G_1 = 3r = ? = 3\times 3 = 9$

$G_2 = 3r^2 = ? = 3.(3)^2 = 3\times 9 = 27$

So $81 = 3r^3$

$\Rightarrow 27 = r^3$

$\Rightarrow 3^3 = r^3$

$3 = r$ ✓

Common Ratio

$3, 9, 27, 81$ G.P.

$\Rightarrow a^{\frac{1}{2}}.b^{\frac{1}{2}} = \dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow a^{n+\frac{1}{2}}.b^{\frac{1}{2}} + a^{\frac{1}{2}}.b^{n+\frac{1}{2}} = a^{n+1}+b^{n+1}$

$\Rightarrow a^{\frac{1}{2}}.b^{n+\frac{1}{2}} - b^{n+1} = a^{n+1} - a^{n+\frac{1}{2}}.b^{\frac{1}{2}}$

$\Rightarrow b^{n+\frac{1}{2}}\cancel{(a^{\frac{1}{2}}-b^{\frac{1}{2}})} = a^{n+\frac{1}{2}}\cancel{(a^{\frac{1}{2}}-b^{\frac{1}{2}})}$

$\Rightarrow b^{n+\frac{1}{2}\rightarrow 0} = a^{n+\frac{1}{2}\rightarrow 0}$

$1 = 1$

$n+\frac{1}{2} = 0$

$n = -\frac{1}{2}$

$\left(\frac{2}{3}\right)^{n+\frac{1}{2}\rightarrow 0}$ … $\frac{n+\frac{1}{2}}{2} = \frac{n+\frac{1}{2}}{3}$ → 1, 1

Q.27 Geometric mean between a & b $= \dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$

Given

$\Rightarrow \sqrt{ab} = \dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow (a.b)^{\frac{1}{2}} = ($ " $)$

Q.28 Let the no. be a & b.

G.M. = $\sqrt{ab}$

ATQ. Sum = 6 × G.M

$\Rightarrow \frac{a+b}{b} = \frac{6\sqrt{ab}}{b}$

$\Rightarrow \frac{a}{b} + 1 = \frac{6\sqrt{a}\cdot\cancel{\sqrt{b}}}{\sqrt{b}\cdot\cancel{\sqrt{b}}}$

$\Rightarrow \frac{a}{b} + 1 = 6\sqrt{\frac{a}{b}}$

Let $\frac{a}{b} = x$

$\Rightarrow x + 1 = 6\sqrt{x}$ Square

$\Rightarrow (x+1)^2 = (6\sqrt{x})^2$

$\Rightarrow x^2 + 1 + 2x = 36 \cdot x$

$\Rightarrow x^2 + 2x - 36x + 1 = 0$

$\Rightarrow 1.x^2 - 34x + 1 = 0$

$\Rightarrow x = \frac{34 \pm \sqrt{(-34)^2 - 4.1.1}}{2}$

$x = \frac{34 \pm \sqrt{1156 - 4}}{2}$

$x = \frac{34 \pm \sqrt{1152}}{2}$

$x = \frac{34 \pm \sqrt{4.4.8.9}}{2}$

$ax^2 + bx + c = 0$

$\downarrow$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

4	1152
4	288
8	72
9	9
	1

$x = \frac{34 \pm 2.2.2\sqrt{2}.3}{2}$

$x = 17 \pm 12\sqrt{2} = \frac{a}{b}$

$a:b = 17 + 12\sqrt{2} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

$a:b = 17 - 12\sqrt{2} = \frac{3-2\sqrt{2}}{3+2\sqrt{2}}$

To Prove ↓

$\boxed{\text{let } x = \frac{a}{b} = a:b = \frac{3+2\sqrt{2}}{3-2\sqrt{2}} = 17+12\sqrt{2}}$

$\frac{3+2\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{(3+2\sqrt{2})^2}{(3)^2 - (2\sqrt{2})^2} = \frac{9+8+12\sqrt{2}}{9-8 \to 1}$

$= 17+12\sqrt{2}$

Q.29 Let 2 positive numbers $= x_1, x_2$

AM of x_1 & $x_2 = A$

$$\boxed{\begin{aligned} x_1 &= ? \\ x_2 &= ? \end{aligned}}$$

$$\Rightarrow \frac{x_1+x_2}{2} = A$$

$$\Rightarrow \boxed{x_1+x_2 = 2A} \quad —(1)$$

GM of x_1 & $x_2 = G$

$$\Rightarrow \sqrt{x_1 \cdot x_2} = G$$

$$\Rightarrow \boxed{x_1 x_2 = G^2} \quad —(2)$$

Quadratic Equation. Roots $= x_1, x_2$

$$x^2 - (\text{Sum of Roots}) \cdot x + (\text{Product of Roots}) = 0$$

$$\Rightarrow x^2 - (x_1+x_2)x + (x_1 \times x_2) = 0$$

$$\Rightarrow x^2 - (2A) \cdot x + (G^2) = 0$$

Roots $= x_1, x_2$

$$1 \cdot x^2 - 2Ax + G^2 = 0 \to x_1, x_2$$

solve (Roots).

Quadratic Form

$$x = \frac{-(-2A) \pm \sqrt{(-2A)^2 - 4 \cdot 1 \cdot G^2}}{2}$$

$$x = \frac{2A \pm \sqrt{4A^2 - 4G^2}}{2}$$

$$x = \frac{\cancel{2}A \pm \cancel{2}\sqrt{A^2 - G^2}}{\cancel{2}}$$

$$x = A \pm \sqrt{(A+G)(A-G)}$$

$$x_1 = A + \sqrt{(A+G) \cdot (A-G)}$$

$$x_2 = A - \sqrt{(A+G) \cdot (A-G)}$$

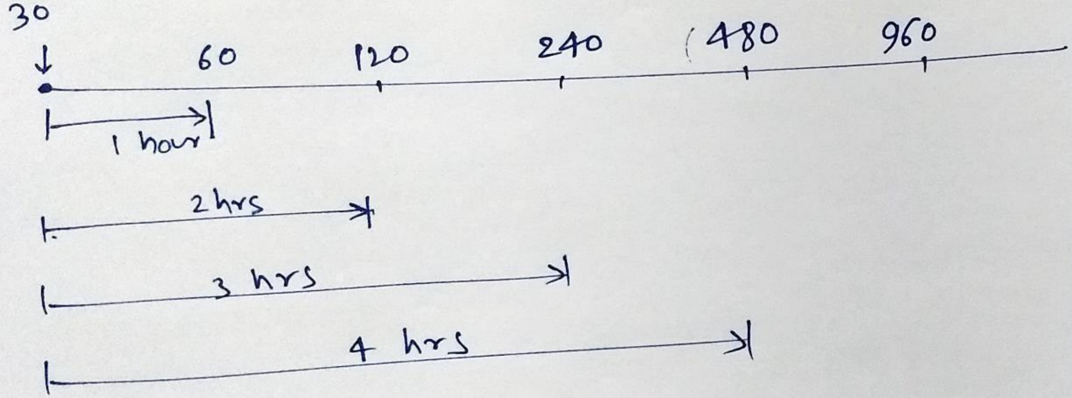
Q.30

Area $\boxed{30}$ ← Culture

Double in one hour

Bacteria

G.P. $\underline{r=2}$



$\underline{\underline{G.P.}}$ a_1 a_2 a_3 a_4 - - -

60 120 240 480 - - -

$\underline{r=2}$ a

$a_n = a \cdot r^{n-1}$

At the end of $\underline{n\text{-hrs}}$. $= a_n = 60 \times (2)^{n-1}$

At the end of

2 hrs $= a_2 = 60 \times (2)^{2-1}$

$= 60 \times 2^1$

$= 120$

At the end of

4 hrs $= 60 \times (2)^{4-1}$

$= 60 \times 2^3$

$= 60 \times 8 = 480$

At the end of n hrs.

$a_n = 60 \times (2)^{n-1}$

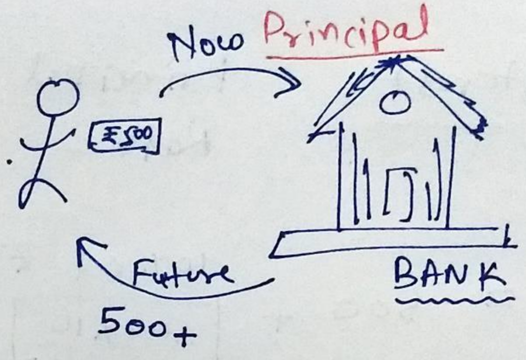
$= 30 \times 2^1 \times 2^{n-1}$

$= 30 \times 2^n$ ✓

Q. 31 Principal = ₹ 500

Rate = 10 %

Time = 10 years



Amount = Principal + S.I.

or

Principal + C.I.

Simple Interest.

P = ₹ 500

R = 10%

₹500 का 10%

After one year = 500 + $\boxed{500 \times \frac{10}{100}}$ = 500 + $\boxed{50}$ = 550

500 का 10% 500 का 10%

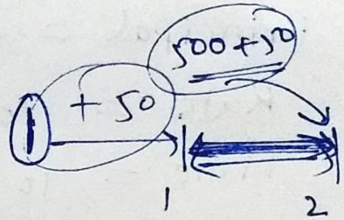
After 2 years = 500 + $\boxed{50}$ + $\boxed{50}$ = 600

After 3 years = $\underbrace{500}_{\text{Principal}}$ + $\underbrace{\boxed{50} + \boxed{50} + \boxed{50}}_{\text{Simple Interest}}$ = $\underbrace{650}_{\text{Amount}}$

Compound Interest.

Principal = 500 Rs.

Rate = 10 %.



After one year = $500 + \boxed{\frac{\cancel{500}\times 10}{\cancel{100}}}$ = 500 + 50 = 550

(10% of ₹500)

After 2 years = $\underbrace{500 + \boxed{50}} + \boxed{\frac{55\cancel{0}\times 1\cancel{0}}{1\cancel{00}}}$ = 500 + 50 + 55 = 605

(10% of ₹550)

After 3 years. = $\underbrace{500 + \boxed{50} + \boxed{55}} + \boxed{\frac{605\times 1\cancel{0}}{10\cancel{0}}}$ = 500 + 50 + 55 + 60.5

(10% of 605; 60.5)

= ₹665.5

Principal = ₹ 500

Rate = 10%.

$\rightarrow P\left(1+\frac{R}{100}\right)^n$

Amount by S.I.	Amount by C.I.	
₹ 550.	a_1 = ₹ 550	← After one year
₹ 600	a_2 = ₹ 605	← After 2 years
₹ 650	a_3 = ₹ 665.5	← After 3 years.
⋮	⋮	

AP. ✗   Our Question ✓   G.P.

$r = \frac{a_2}{a_1} = \frac{\cancel{605}^{\cancel{55}\,11}}{\cancel{550}_{\cancel{50}\,10}} = \frac{11}{10} = 1.1$

$\frac{665.5}{605}$

Amount $\underset{a_1}{550}, \underset{a_2}{605}, \underset{a_3}{665.5} \text{------} \underset{a_{10}}{}$

$r = 1.1$

$a_{10} = a \cdot r^{10-1}$

$= 550 \times (1.1)^9 = 500 \times (1.1)^{10}$ — $\underline{\underline{500 \times (1.1) \times (1.1)^9}}$

Q.32 Roots of Quadratic eqn. = x_1 & x_2 (let)

AM of Roots = 8 = $\frac{x_1+x_2}{2}$ $\Rightarrow$ $\boxed{x_1+x_2=16}$

GM of Roots = 5 = $\sqrt{x_1 \cdot x_2}$ $\Rightarrow$ $\boxed{x_1 \cdot x_2 = 25}$

<u>Quadratic eqn.</u>

$$x^2 - (\text{Sum of Roots})\cdot x + \left(\begin{array}{c}\text{product}\\ \text{of}\\ \text{roots}\end{array}\right) = 0$$

$$\Rightarrow \quad x^2 - (x_1 + x_2)\cdot x + x_1 \cdot x_2 = 0$$

$$\Rightarrow \quad \boxed{x^2 - 16x + 25 = 0}$$

Ex 8.4

Sum of n-terms of special series

① $1+2+3+\dots+n$ = Sum of first n-natural numbers $= \sum_{K=1}^{n} K = \frac{n(n+1)}{2}$

② $1^2+2^2+3^2+\dots+n^2$ = Sum of squares of first n-natural numbers $= \sum_{K=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$

③ $1^3+2^3+3^3+\dots+n^3$ = Sum of cubes of first n-natural numbers $= \sum_{K=1}^{n} K^3 = \left[\frac{n(n+1)}{2}\right]^2$

Note: (I) Sum of the n-terms of any series

$= S_n = a_1 + a_2 + \dots + a_n$

$$\boxed{S_n = \sum_{K=1}^{n} a_K}$$

(II) $\sum_{K=1}^{n} (a_K \pm t_K) = \sum_{K=1}^{n} a_K \pm \sum_{K=1}^{n} t_K$

$\sum_{K=1}^{n} 3 \cdot a_K = 3 \cdot \sum_{K=1}^{n} a_K$ Right

$\sum_{K=1}^{n} a_K \cdot t_K = \sum_{K=1}^{n} a_K \cdot \sum_{K=1}^{n} t_K$ wrong.

(e.g.) $(11^3 + 12^3 + 13^3 + \dots + 20^3)$ $\quad \sum_{K=1}^{n} K^3 = \left[\frac{n(n+1)}{2}\right]^2$

$$= \boxed{(1^3 + 2^3 + \dots + 10^3) + (11^3 + 12^3 + \dots + 20^3)} - (\underline{1^3 + 2^3 + \dots + 10^3})$$

$$= \left[\frac{\overset{10}{\cancel{20}} \cdot (20+1)}{\cancel{2}}\right]^2 - \left[\frac{\overset{5}{\cancel{10}} \cdot (10+1)}{\cancel{2}}\right]^2$$

$$= (10 \times 21)^2 - (5 \times 11)^2$$

$$= (210)^2 - (55)^2 = 44100 - 3025$$

$$= 41075$$

(e.g.) Find the sum of n-terms of series $\underline{(1)} + \underline{(1+2)} + \underline{(1+2+3)} + \dots + (\quad)$ a_n

$$a_n = (1 + 2 + \dots + n) = \frac{n(n+1)}{2}$$

$$a_n = \frac{n^2 + n}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$a_n = \frac{n^2}{2} + \frac{n}{2}$$

$$S_n = \sum_{K=1}^{n} (a_K)$$

$$S_n = \sum_{K=1}^{n} \left(\frac{K^2}{2} + \frac{K}{2}\right)$$

$$= \sum_{K=1}^{n} \frac{K^2}{2} + \sum_{K=1}^{n} \frac{K}{2}$$

$$= \frac{1}{2} \sum_{K=1}^{n} K^2 + \frac{1}{2} \sum_{K=1}^{n} K$$

$$= \frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{1}{2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$= \underline{\qquad\qquad}$$

★ **Example : 19** Find the sum of n-terms of the series : $5+11+19+29+41 \ldots\ldots$ $\quad S_n = ?$

Difference$_1$ → 6 8 10 12 ← AP / GP

Diff.$_2$ → 2 2 2

Today's Concept

$$S_n = \overbrace{5 + \underbrace{11 + 19 + 29 + 41 + \ldots + a_{n-1} + a_n}_{(n-1)}}^{n}$$

$$S_n = \quad 5 + 11 + 19 + 29 + \ldots\ a_{n-2} + a_{n-1} + a_n$$

$$0 = \underbrace{5}\ + \underbrace{6 + 8 + 10 + 12 + \ldots + (\ ) + (\ )}_{AP.} - a_n$$

First term $= a = 6$

C.D. $= 2 = d$

no. of terms $= (n-1)$

$$\text{Sum} = S^{AP}_{n-1} = \frac{n-1}{2}\left\{2\times 6 + (n-2).2\right\}$$

$$= \frac{n-1}{\cancel{2}} \cdot \left\{\cancel{12}^{\,6} + \cancel{2n}^{\,n} - \cancel{4}^{\,2}\right\}$$

$$= (n-1)\cdot(n+4) = n^2+3n-4$$

$\Rightarrow 0 = 5 + (n^2+3n-4) - a_n$

$\Rightarrow \boxed{a_n = n^2+3n+1}$

$$a_n = n^2 + 3n + 1$$

$$S_n = \sum_{K=1}^{n} a_K = \sum_{K=1}^{n} (K^2 + 3K + 1)$$

$1 = 1 \times 1$

$$S_n = \sum_{K=1}^{n} K^2 + 3 \cdot \sum_{K=1}^{n} K + \sum_{K=1}^{n} 1$$

$$\sum_{K=1}^{n} 1 = \underbrace{1 + 1 + 1 + \cdots + 1}_{n\text{-times}} = n$$

$$S_n = \frac{\boxed{n}(n+1)(2n+1)}{6} + 3 \cdot \frac{\boxed{n}(n+1)}{2} + \boxed{n}$$

$$S_n = n \left\{ \frac{2n^2 + 3n + 1}{6} + \frac{3n+3}{2} + \frac{1}{1} \right\}$$

$$S_n = n \cdot \left\{ \frac{2n^2 + 3n + 1 + 9n + 9 + 6}{6} \right\} = n \cdot \left\{ \frac{\overset{n^2 + 6n + 8}{\cancel{2n^2 + 12n + 16}}}{\underset{3}{\cancel{6}}} \right\}$$

$$S_n = \frac{n(n^2 + 6n + 8)}{3} = \frac{n(n+2)(n+4)}{3}$$

Q.1 $\underbrace{1\times2}_{a_1} + \underbrace{2\times3}_{a_2} + \underbrace{3\times4}_{a_3} + \underbrace{4\times5}_{a_4} + \dots$

n^{th} term $= a_n = n.(n+1)$

$a_n = n^2 + n$

Sum of n-terms $= S_n = \sum_{K=1}^{n} a_K$

$$S_n = \sum_{K=1}^{n} (K^2+K)$$

$$= \sum_{K=1}^{n} K^2 + \sum_{K=1}^{n} K$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} \cdot \left\{\frac{2n+1}{3} + 1\right\} = \frac{n(n+1)\overset{(n+2)}{\cancel{(2n+4)}}}{\cancel{2}\times3}$$

Q.2 $\underbrace{1\times2\times3}_{a_1} + \underbrace{2\times3\times4}_{a_2} + \underbrace{3\times4\times5}_{a_3} + \dots$

n^{th} term $= \boxed{a_n = n.(n+1).(n+2)}$

$$S_n = \sum_{K=1}^{n} a_K = \sum_{K=1}^{n} K(K+1)(K+2)$$

$$S_n = \sum_{K=1}^{n} K(K^2+3K+2)$$

$$S_n = \sum_{K=1}^{n} (K^3+3K^2+2K)$$

$$S_n = \sum_{K=1}^{n} K^3 + 3\sum_{K=1}^{n} K^2 + 2\sum_{K=1}^{n} K$$

$$S_n = \left[\frac{n(n+1)}{2}\right]^2 + 3\cdot\frac{(n)(n+1)(2n+1)}{6} + 2\cdot\frac{n(n+1)}{2}$$

Q.3

$$\underbrace{3\times1^2}_{a_1}+\underbrace{5\times2^2}_{a_2}+\underbrace{7\times3^2}_{a_3}+\cdots$$

odd no.

$$\boxed{3,5,7,\ldots}$$

$2n-1$ या $2n+1$

1, 3, 5, ⋮ — 3, 5, 7, ⋮

n^{th} term $= a_n = (2n+1)\times n^2$

$$\boxed{a_n = 2n^3+n^2}$$

$$S_n = \sum_{K=1}^{n} a_K = \sum_{K=1}^{n}(2K^3+K^2)$$

$$S_n = 2\sum_{K=1}^{n}K^3 + \sum_{K=1}^{n}K^2$$

$$S_n = 2\cdot\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \cancel{2}\cdot\frac{n^2\cdot(n+1)^2}{\cancel{4}\,2} + \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{n(n+1)}{2}\left\{\frac{n(n+1)}{1} + \frac{2n+1}{3}\right\}$$

$$S_n = \frac{n(n+1)}{2}\left\{\frac{3(n^2+n)+2n+1}{3}\right\}$$

$$= \frac{n(n+1)}{2}\cdot\left\{\frac{3n^2+3n+2n+1}{3}\right\}$$

$$= \frac{n(n+1)}{2}\cdot\left(\frac{3n^2+5n+1}{3}\right)$$

$$= \frac{n(n+1)\cdot(3n^2+5n+1)}{6}$$

Q.4 $\underbrace{\frac{1}{1\times 2}}_{a_1} + \underbrace{\frac{1}{2\times 3}}_{a_2} + \underbrace{\frac{1}{3\times 4}}_{a_3} + \dots$ upto n-terms

n^{th} term $= a_n = \frac{1}{n\times(n+1)} = \frac{1}{n(n+1)}$

Sum of n-terms $= \boxed{S_n = \sum_{K=1}^{n} a_K}$

$\Rightarrow S_n = \sum_{K=1}^{n} \frac{1}{K.(K+1)}$

$\underset{\text{first}}{K} \quad \underset{\text{last.}}{(K+1)}$

Difference of first & last $= (K+1) - K$

$= 1 \rightarrow$ Numerator

$\Rightarrow S_n = \sum_{K=1}^{n} \frac{(K+1)-K}{K.(K+1)}$

$S_n = \sum_{K=1}^{n} \frac{(K+1)-K}{K.(K+1)}$

$= \sum_{K=1}^{n} \left\{ \frac{1}{K} - \frac{1}{K+1} \right\}$.

$= \left\{ \frac{1}{1} - \cancel{\frac{1}{2}} \right\}$

$+ \left\{ \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} \right\}$

$+ \left\{ \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} \right\}$

$\vdots$

$+ \left\{ \cancel{\frac{1}{n}} - \frac{1}{n+1} \right\}$

$= 1 - \frac{1}{n+1}$

$= \frac{(n+\cancel{1})-\cancel{1}}{n+1} = \frac{n}{n+1}$

Q. 5 $\quad \left[5^2+6^2+7^2+\dots+20^2\right]$

$$= \left(1^2+2^2+3^2+4^2\right) + \left[5^2+6^2+\dots+20^2\right] - \left(1^2+2^2+3^2+4^2\right)$$

$$= \left(1^2+2^2+\dots+20^2\right) - \underline{\left(1^2+2^2+\dots+4^2\right)}$$

$$= \frac{20\cdot(20+1)(20\times 2+1)}{6} - \frac{4\cdot(4+1)\cdot(2\times 4+1)}{6}$$

$$= \frac{\cancel{20}^{10}\times\cancel{21}^{7}\times 41}{\cancel{6}\,\cancel{3}} - \frac{{}^{2}\cancel{4}\times 5\times\cancel{9}\,3}{\cancel{6}\,\cancel{3}}$$

$$= 2870 - 30$$

$$= 2840$$

Q.6 $\underbrace{3\times 8}_{a_1} + \underbrace{6\times 11}_{a_2} + \underbrace{9\times 14}_{a_3} + \cdots + \underbrace{(3n)\cdot(3n+5)}_{a_n}$

3, 6, 9, ..., 3n

1 2 3 n

8, 11, 14, ... A_n,

+3 +3

AP

$A_n = a + (n-1)\cdot d$

$8 + (n-1)\cdot 3 = 8 + 3n - 3$

$= 3n + 5$

n^{th} - term $= a_n = 3n\cdot(3n+5) = 9n^2 + 15n$

$$S_n = \sum_{K=1}^{n} a_K = \sum_{K=1}^{n} (9K^2 + 15K)$$

$$S_n = 9\sum_{K=1}^{n} K^2 + 15\sum_{K=1}^{n} K$$

$$= 9\cdot\frac{n(n+1)(2n+1)}{6} + 15\,\frac{n\cdot(n+1)}{2}$$

$$S_n = \frac{3\cdot n(n+1)}{2}\cdot\left\{\frac{\cancel{3}(2n+1)}{\cancel{3}} + 5\right\}$$

$$S_n = \frac{3n(n+1)}{\cancel{2}}\cdot\cancel{(2n+6)}\ \ n+3$$

$$S_n = 3\cdot n\cdot(n+1)\cdot(n+3)$$

Q.7 $\underbrace{(1^2)}_{a_1}+\underbrace{(1^2+2^2)}_{a_2}+\underbrace{(1^2+2^2+3^2)}_{a_3}+\dots+\underbrace{(1^2+2^2+\dots+n^2)}_{a_n}$

$$a_n=(1^2+2^2+3^2+\dots+n^2)=\frac{n(n+1)(2n+1)}{6}=\frac{n(2n^2+3n+1)}{6}=\frac{2n^3}{6}+\frac{3n^2}{6}+\frac{n}{6}$$

$$\boxed{a_n=\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}}$$

$$\boxed{S_n=\sum_{K=1}^{n} a_K}$$

$$S_n=\sum_{K=1}^{n}\left(\frac{K^3}{3}+\frac{K^2}{2}+\frac{K}{6}\right)$$

$$S_n=\frac{1}{3}\sum_{K=1}^{n}K^3+\frac{1}{2}\sum_{K=1}^{n}K^2+\frac{1}{6}\sum_{K=1}^{n}K$$

$$S_n=\frac{1}{3}\cdot\left(\frac{n(n+1)}{2}\right)^2+\frac{1}{2}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{1}{6}\cdot\frac{n(n+1)}{2}$$

$$=\frac{n^2\cdot(n+1)^2}{12}+\frac{n(n+1)(2n+1)}{12}+\frac{n(n+1)}{12}$$

$$S_n=\frac{n(n+1)}{12}\cdot\{n(n+1)+(2n+1)+1\}$$

$$S_n=\frac{n(n+1)}{12}\cdot\{n^2+n+2n+1+1\}$$

$$S_n=\frac{n(n+1)}{12}\cdot\{n^2+3n+2\}$$

$$S_n=\frac{n(n+1)\cdot(n+1)(n+2)}{12}$$

$$\boxed{S_n=\frac{n(n+1)^2\cdot(n+2)}{12}}$$

Q.8 $\underline{n^{th}\text{ - term}} = a_n = n(n+1)(n+4) = n(n^2+5n+4)$

$= n^3+5n^2+4n$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} (k^3+5k^2+4k)$$

$$S_n = \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$S_n = \left[\frac{n(n+1)}{2}\right]^2 + 5\left(\frac{n(n+1)(2n+1)}{6}\right) + \cancel{4}^{2}\left(\frac{n(n+1)}{\cancel{2}}\right)$$

$$S_n = n(n+1)\left\{\frac{n(n+1)}{4} + \frac{5(2n+1)}{6} + \frac{2}{1}\right\}$$

$$S_n = n(n+1)\left\{\frac{3n^2+3n+20n+10+24}{12}\right\}$$

$3\times2\ 17$

3×34

$$S_n = \frac{n(n+1)\cdot(3n^2+23n+34)}{12}$$

$$S_n = \frac{n(n+1)\cdot(3n^2+6n+17n+34)}{12}$$

$$S_n = \frac{n(n+1)\left[3n(n+2)+17(n+2)\right]}{12}$$

$$S_n = \frac{n(n+1)(n+2)(3n+17)}{12}$$

Q.9 n^{th} term $= a_n = n^2 + 2^n$

$$S_n = \sum_{K=1}^{n} a_K = \sum_{K=1}^{n} (K^2 + 2^K) = \underbrace{\sum_{K=1}^{n} (K^2)}_{\frac{n(n+1)(2n+1)}{6}} + \underbrace{\sum_{K=1}^{n} (2^K)}$$

$$\sum_{K=1}^{n} (2^K) = (2^1) + (2^2) + \cdots \cdots + (2^n)$$

$$= \boxed{2^1 + 2^2 + 2^3 + \cdots + 2^n} \text{ G.P.}$$

$$= \frac{a.(r^n - 1)}{r-1}$$

$a = 2$
$r = 2$
no. of terms $= n$

$$= \frac{2.(2^n - 1)}{2-1} \quad (2-1 \to 1)$$

$$= \frac{2^1.(2^n - 1)}{}$$

$$= 2^{n+1} - 2$$

$$\underline{S_n = \frac{n(n+1)(2n+1)}{6} + 2^{n+1} - 2}$$

Q. 10 n^{th} - term $= a_n = (2n-1)^2$

$(a-b)^2 = a^2+b^2-2ab$

$\Rightarrow a_n = (2n)^2 + (1)^2 - 2(2n)(1)$

$\Rightarrow \boxed{a_n = 4n^2 + 1 - 4n}$

Sum of n-terms $= S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} (4k^2 + 1 - 4k)$

$S_n = 4\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1 - 4\sum_{k=1}^{n} k$

$\sum_{k=1}^{n} 1 = \underbrace{1+1+\cdots+1}_{n\text{-times}} = n$

$S_n = \cancel{4}^{2} \cdot \frac{n(n+1)(2n+1)}{\cancel{6}_{3}} + n - \cancel{4}^{2} \cdot \frac{n(n+1)}{\cancel{2}}$

$S_n = n\left\{ \frac{2}{3}(2n^2+3n+1) + \frac{1}{1} - \frac{2(n+1)}{1} \right\}$

$S_n = n \cdot \left\{ \frac{4n^2 + \cancel{6n} + 2 + 3 - \cancel{6n} - 6}{3} \right\}$

$\Rightarrow S_n = \frac{n \cdot (4n^2 - 1)}{3}$

$S_n = \frac{n \cdot [(2n)^2 - 1^2]}{3}$

$\boxed{S_n = \frac{n(2n+1)(2n-1)}{3}}$

Miscellaneous Exercise 8.5

Q.1 A.P.

To prove:

$$\boxed{(m+n)^{th}\text{ term} + (m-n)^{th}\text{ term} = 2(m^{th}\text{ term})}$$

Proof: first term $= a$ (let) $\boxed{a_n = a+(n-1).d}$

C.D. $= d$

LHS $= (m+n)^{th}$ term $+ (m-n)^{th}$ term

$= a_{m+n} + a_{m-n}$

$= a + (m+n-1).d + a + (m-n-1).d$

$= 2a + (m+\cancel{n}-1+m-\cancel{n}-1).d$

$= 2a + (2m-2).d$

$= 2a + 2(m-1).d = 2\{a+(m-1).d\}$

$= 2\{a_m\}$

$= 2(m^{th}$ term$)$

$=$ RHS.

Q.2 Sum of 3 No. in AP $= 24$

their Product $= 440$.

Let the numbers be

$a-d,\ a,\ a+d,$

ATQ. $(a-\cancel{d})+(a)+(a+\cancel{d}) = 24$

$\Rightarrow \cancel{3}a = \cancel{24}\ 8$

$\boxed{a=8}$

No. $\rightarrow$ $8-d,\ 8,\ 8+d$

ATQ. $(8-d).\cancel{8}.(8+d) = \cancel{440}\ 55$

$\Rightarrow 8^2 - d^2 = 55$

$\Rightarrow 64 - d^2 = 55$

$\Rightarrow \boxed{9 = d^2} \Rightarrow \boxed{d = \pm 3}$

$a = 8$, $d = +3$ (you can also take $-3 = d$)

No. $a - d = 5$
$a = 8$
$a + d = 11$

11
8
5

Q.3 AP. Let ($a_1 = a$, $CD = d$)

Sum of n terms $= S_n = S_1 = \frac{n}{2}(2a + (n-1).d)$

Sum of 2n terms $= S_{2n} = S_2 = \frac{2n}{2}[2a + (2n-1).d]$

Sum of 3n terms $= S_{3n} = S_3 = \frac{3n}{2}[2a + (3n-1).d]$

To Prove: $S_3 = 3(S_2 - S_1)$

$RHS = 3(S_2 - S_1)$

$= 3\left\{\frac{2n}{2}\left(2a + (2n-1).d\right) - \frac{n}{2}[2a + (n-1).d]\right\}$

$= 3\frac{n}{2}.\{4a + 4nd - 2d - 2a - nd + d\}$

$= \frac{3n}{2}.(2a + 3nd - d)$

$= \frac{3n}{2}.\{2a + (3n-1)d\}$

$= S_3$

$= LHS$.

Q.4 between 200 & 400

Divisible by 7

AP. $n = ?$

AP Sequence: $\longrightarrow$ $(203, 210, 217, \ldots, 399)$ $\to a = 203$, $\to d = 7$

$S_n = ?$

$$7 \overline{)200} (28$$
$$-14$$
$$60$$
$$-56$$
$$4$$

$$7 \overline{)400} (57$$
$$-35$$
$$50$$
$$-49$$
$$1$$

$200 = 7 \times 28 + 4$ $\qquad$ $400 = 7 \times 57 + 1$

$200 - 4 = 196$, $196 + 7 = 203$ $\qquad$ $400 - 1 = 399$

let $399 = n^{th}$ term

$399 = a_n$

$\Rightarrow 399 = a + (n-1) \cdot d$

$\Rightarrow 399 = 203 + (n-1) 7$

$\Rightarrow \cancel{196} = (n-1) \cdot \cancel{7}$
28

$\Rightarrow 28 = n - 1 \Rightarrow 29 = n$

$$S_n = \frac{n}{2}\left(2a + (n-1) \cdot d\right)$$

$$S_{29} = \frac{29}{\cancel{2}} \cdot \left(\cancel{2} \times 203 + \overset{14}{\cancel{28}} \cdot 7\right)$$

$$S_{29} = 29 \cdot (203 + 98) = 29(301) = 8729 \checkmark$$

Q.5 Integers from 1 to 100 Sum = ?

Divisible by '2' or '5'

Main Sequence → 2, 4, 5, 6, 8, 10, 12, ...

Actual

$S_n = \frac{n}{2}(2a + (n-1) \cdot d)$

Divisibly by '2' → 2, 4, 6, 8, 10, 12, ... 96, 98, 100 → $Sum_1 = S_1 = \frac{50}{2}(4 + 49 \times 2)$

Divisibly by '5' → 5, 10, 15, 20, 25, ... 90, 95, 100 → $S_2 = \frac{20}{2}(10 + 19 \times 5)$

Common No. → Divisibly by '2' & '5' = 10 → 10, 20, 30, ... 100 → $S_3 = \frac{10}{2}(20 + 9 \times 10)$

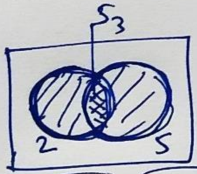
Required Answer $= S_1 + S_2 - S_3$

$= 25(4 + 98) + 10(10 + 95) - 5(20 + 90)$

$= 25(102) + 10(105) - 5(110)$

$= 2550 + 1050 - 550$

$= 3050$



$S_1 + S_2 - S_3$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Q.6 Sum of all two Digit Numbers = ?

Divisor = 4 then remainder = 1

$\boxed{10 \longleftrightarrow 99}$

10, 11, 12, 13, 14, ..., 96, 97, 98, 99, 100

r=1   r=1

4 Divisible   4 Divisible   4 Divisible

Required sequence : → 13, 17, 21, ..., 97,

Sum = ?

$a = 13$

$d = 4$

$n = ? = 22$

$$\boxed{S_n = \frac{n}{2}\left(2a + (n-1).d\right)}$$

let $97 = a_n$

$\Rightarrow 97 = a + (n-1).d$

$\Rightarrow 97 = 13 + (n-1).4$

$\Rightarrow \cancel{84} = (n-1).\cancel{4}$

$21 = n-1$

$\Rightarrow 22 = n$

$$S_n = S_{22} = \frac{\overset{11}{\cancel{22}}}{\cancel{2}}(26 + 21 \times 4)$$

$$S_{22} = 11(26 + 84)$$

$$= 11(110) = 1210$$

$\boxed{Q.7}$ $\boxed{f(x+y) = f(x).f(y)}$ $(x, y \in N)$

$a = 3$, $r = 3$,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$f(1) = 3$, $\sum_{x=1}^{n} f(x) = 120$, $n = ?$

$\Rightarrow \boxed{f(1) + f(2) + f(3) + \ldots + f(n) = 120} \Rightarrow \underbrace{3 + 3^2 + 3^3 + \ldots + 3^n = 120}$ sum

G.P.

$f(1) \to 3$, $f(2) \to 3^2$, $f(3) \to 3^3$, $f(n) \to 3^n$

$x = 1 = y$

$f(2) = f(1+1) = f(1).f(1) = 3 \times 3 = 9 = 3^2$

$f(3) = f(1+2) = f(1).f(2) = 3 \times 9 = 3^3$

$f(4) = 3^4$

Similarly, $\boxed{f(n) = 3^n}$

$$\Rightarrow \frac{\cancel{3}(3^n - 1)}{3 - 1} = \cancel{120}\ 40$$

$\Rightarrow 3^n - 1 = 80$

$\Rightarrow 3^n = 81 = 3^4$

$n = 4$

Q.8 Sum of $\underbrace{\text{some}}_{n \text{ let}}$ terms of G.P. = 315

$a = 5$, $r = 2$, $\underbrace{\text{last term}}_{a_n = ?}$, $\underbrace{\text{No. of terms}}_{n = ? = 6}$

ATQ

$S_n = 315$

$\Rightarrow \frac{a(r^n - 1)}{r-1} = 315$

$\Rightarrow \frac{\cancel{5}(2^n - 1)}{2-1} = \cancel{315}\ 63$

$\Rightarrow 2^n - 1 = 63$

$\Rightarrow 2^n = 64 = 2^6$

$\boxed{n = 6}$

$a_n = a \cdot r^{n-1}$

$= 5 \times (2)^{6-1}$

$= 5 \times 2^5$

$= 5 \times 32$

$= 160$

Q.9 $a = 1$ $r = ?$

$a_3 + a_5 = 90$

$\Rightarrow a r^2 + a r^4 = 90$

$\Rightarrow 1 \cdot r^2 + 1 \cdot r^4 = 90$

$\Rightarrow r^2 + r^4 - 90 = 0$

$\Rightarrow (r^2)^2 + (r^2) - 90 = 0$

$r^2 = x$ let

then $x^2 + x - 90 = 0$

$\Rightarrow x^2 + 10x - 9x - 90 = 0$

$\Rightarrow x(x+10) - 9(x+10) = 0$

$\Rightarrow (x+10)(x-9) = 0$

$x = -10$ $x = 9$

reject $r^2 = -10$ $r^2 = 9$

$\boxed{r = \pm 3}$

$r = \pm 3$

Q.10 Sum of 3-No. in G.P. $= 56 = a + ar + ar^2$ $\Rightarrow$ $\boxed{56 = a(1+r+r^2)}$ — (1)

(-1), (-7), (-21)

$a = ?$
$r = ?$

Let $a, ar, ar^2 \leftarrow$ GP.

$a-1, ar-7, ar^2-21 \rightarrow$ AP.

Note: If a, b, c are in AP.
then $2b = a+c$

$\therefore 2(ar-7) = (a-1) + (ar^2-21)$

$\Rightarrow 2ar - 14 = a - 1 + ar^2 - 21$

$\Rightarrow 22 - 14 = a + ar^2 - 2ar$

$\Rightarrow \boxed{8 = a(1+r^2-2r)}$ — (2)

By $\dfrac{\text{eq}^n (1)}{\text{eq}^n (2)}$:→

$\Rightarrow \dfrac{\cancel{56}^{7}}{\cancel{8}} = \dfrac{\cancel{a}(1+r+r^2)}{\cancel{a}(1+r^2-2r)}$

$\Rightarrow 7 = \dfrac{1+r+r^2}{1+r^2-2r}$

$\Rightarrow 7 + 7r^2 - 14r = 1 + r + r^2$

$\Rightarrow (6r^2 - 15r + 6 = 0)/3$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow 2r^2 - 4r - r + 2 = 0$

$\Rightarrow 2r(r-2) - (r-2) = 0$

$\Rightarrow (2r-1)(r-2) = 0$

$r = \frac{1}{2}, \quad r = 2$

$\underline{eq^n.\ (1)}$ $56 = a(1+r+r^2)$

$\underline{r = \frac{1}{2}}$

$56 = a\left(1+\frac{1}{2}+\frac{1}{4}\right)$

$56 = a\left(\frac{4+2+1}{4}\right)$

$\cancel{56}^{8} = a.\frac{\cancel{7}}{4}$

$\boxed{a = 32}$

Numbers

$a = 32$

$ar = 32 \times \frac{1}{2} = 16$

$ar^2 = 32 \times \frac{1}{4} = 8$

32, 16, 8

$\underline{r = 2}$

$\cancel{56}^{8} = a\cancel{(1+2+4)}$

$a = 8$

Numbers

$a = 8$

$ar = 8 \times 2 = 16$

$ar^2 = 8 \times 4 = 32$

8, 16, 32

$\boxed{Q.11}$ Total even no. of terms $= \underline{2n}$

G.P.

$a_1,\ a_2,\ a_3,\ a_4,\ a_5\ \text{-----}\ ,\ a_{2n-1},\ a_{2n}$

$a,\ ar,\ ar^2,\ ar^3,\ ar^4\ \text{----},\ a.r^{2n-2},\ ar^{2n-1}$

ATQ.

$$\underbrace{(a_1+a_2+\dots+a_{2n})}_{G.P.}^{\text{all}} = 5.\underbrace{(a_1+a_3+a_5+\dots+a_{2n-1})}_{G.P.}^{\text{odd}}$$

$$\Rightarrow (a+ar+\dots+ar^{2n-1}) = 5.(a+ar^2+ar^4+\dots+ar^{2n-2})$$

G.P. $a_1 = a$

C.R. $= r$

No. of terms $= 2n$

G.P. $a_1 = a$

CR $= r^2$

No. of terms $= n$

$$\Rightarrow \frac{\cancel{a}(\cancel{r^{2n}-1})}{r-1} = 5 \times \frac{\cancel{a}(\cancel{(r^2)^n-1})}{r^2-1}$$

$\Rightarrow \frac{1}{r-1} = 5 \times \frac{1}{r^2-1^2}$

$\Rightarrow \frac{1}{\cancel{r-1}} = \frac{5}{\cancel{(r-1)}(r+1)}$

$\Rightarrow 1 = \frac{5}{r+1}$

$\Rightarrow r+1 = 5 \Rightarrow \boxed{r=4}$

$\boxed{Q. 12}$ AP. no. of terms $= n = ?$

AP $\rightarrow \underbrace{a_1, a_2, a_3, a_4}_{56}, a_5, \ldots, \underbrace{a_{n-3}, a_{n-2}, a_{n-1}, a_n}_{112}$

$11 = a$

ATQ. $a_1 + a_2 + a_3 + a_4 = 56$

$\Rightarrow a + (a+d) + (a+2d) + (a+3d) = 56$

$\Rightarrow 4a + 6d = 56$

$4a + 6d = 56$

$a = 11$

$\Rightarrow 4 \times 11 + 6d = 56$

$\Rightarrow 44 + 6d = 56$

$\Rightarrow 6d = 12$

$\boxed{d = 2}$

ATQ.

$a_{n-3} + a_{n-2} + a_{n-1} + a_n = 112$

$\Rightarrow a + (n-4) \cdot d + a + (n-3) \cdot d + a + (n-2) \cdot d + a + (n-1) \cdot d = 112$

$\Rightarrow 4a + d\{4n - 10\} = 112$

$\Rightarrow 44 + 2(4n-10) = 112$

$\Rightarrow 44 + 8n - 20 = 112$

$\Rightarrow 8n = 112 - 24$

$\Rightarrow 8n = 88$

$\Rightarrow n = 11$

Q.13

To Prove $\boxed{a, b, c, d \text{ are in G.P.}}$

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

Given $\dfrac{a+bx}{a-bx} = \dfrac{b+cx}{b-cx} = \dfrac{c+dx}{c-dx}$

Proof: I - method:

Componendo - Dividendo

$$\frac{x}{y} = \frac{z}{w} \Rightarrow \frac{x+y}{x-y} = \frac{z+w}{z-w}$$

By applying Componendo - Dividendo

$$\frac{(a+\cancel{bx}) + (a-\cancel{bx})}{(\cancel{a}+bx) - (\cancel{a}-bx)} = \frac{(b+\cancel{cx}) + (b-\cancel{cx})}{(\cancel{b}+cx) - (\cancel{b}-cx)} = \frac{(c+\cancel{dx}) + (c-\cancel{dx})}{(\cancel{c}+dx) - (\cancel{c}-dx)}$$

$$\Rightarrow \frac{\cancel{2}a}{\cancel{2}b\cancel{x}} = \frac{\cancel{2}b}{\cancel{2}c\cancel{x}} = \frac{\cancel{2}c}{\cancel{2}d\cancel{x}} \Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Reciprocal

$$\Rightarrow \boxed{\frac{b}{a} = \frac{c}{b} = \frac{d}{c}}$$

II - method:

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

$+1 \quad +1 \quad +1$

$$\Rightarrow \frac{a+bx}{a-bx} + 1 = \frac{b+cx}{b-cx} + 1 = \frac{c+dx}{c-dx} + 1$$

$$\Rightarrow \frac{a+\cancel{bx}+a-\cancel{bx}}{a-bx} = \frac{b+\cancel{cx}+b-\cancel{cx}}{b-cx} = \frac{c+\cancel{dx}+c-\cancel{dx}}{c-dx}$$

$$\Rightarrow \frac{\cancel{2}a}{a-bx} = \frac{\cancel{2}b}{b-cx} = \frac{\cancel{2}c}{c-dx}$$

$$\Rightarrow \frac{a-bx}{a} = \frac{b-cx}{b} = \frac{c-dx}{c}$$

$-1 \quad -1 \quad -1$

$$\Rightarrow \frac{\cancel{a}-bx-\cancel{a}}{a} = \frac{\cancel{b}-cx-\cancel{b}}{b} = \frac{\cancel{c}-dx-\cancel{c}}{c}$$

$$\Rightarrow \frac{-b\cancel{x}}{a} = \frac{-c\cancel{x}}{b} = \frac{-d\cancel{x}}{c} \Rightarrow \boxed{\frac{b}{a} = \frac{c}{b} = \frac{d}{c}}$$

Q.14 GP $\to a, ar, ar^2, \ldots, a\cdot r^{n-1}$

$a_1 \quad a_2 \quad a_3 \quad a_n$

$S = \text{Sum} = \dfrac{a(r^n-1)}{r-1}$

$P = \text{Product} = (a)\times(ar)\times(ar^2)\times\cdots\times(ar^{n-1})$

$= a^n \times r^{\underbrace{1+2+3+\cdots+(n-1)}_{AP}}$

AP $\longrightarrow S_n = \frac{n}{2}(2a+(n-1)d)$

$= \frac{n}{2}(a+l)$

$= a^n \cdot r^{\frac{(n-1)}{2}\cdot(1+n-1)}$

$P = a^n \cdot r^{\left(\frac{n-1}{2}\right)\cdot n} = a^n \cdot r^{\frac{n(n-1)}{2}}$

$R = \text{Sum of Reciprocals} = \underbrace{\dfrac{1}{a} + \dfrac{1}{ar} + \dfrac{1}{ar^2} + \cdots + \dfrac{1}{ar^{n-1}}}_{G.P.}$

First $= \dfrac{1}{a}$, C.R. $= \dfrac{\frac{1}{ar}}{\frac{1}{a}} = \dfrac{1}{r}$

$R = \dfrac{\frac{1}{a}\left(\left(\frac{1}{r}\right)^n - 1\right)}{\left(\frac{1}{r}-1\right)} = \dfrac{\frac{1}{a}\cdot\left(\frac{1}{r^n}-1\right)}{\left(\frac{1-r}{r}\right)} = \dfrac{\left(\frac{1-r^n}{r^n}\right)}{a\left(\frac{1-r}{r}\right)} = \dfrac{(1-r^n)}{a(1-r)\cdot r^{n-1}}$

To Prove:

$P^2 R^n = S^n$

$\Rightarrow \left[a^n \cdot r^{\frac{n(n-1)}{2}}\right]^2 \cdot \left[\dfrac{(1-r^n)}{a(1-r)\cdot r^{n-1}}\right]^n = \left[\dfrac{a(r^n-1)}{r-1}\right]^n$

$\Rightarrow \dfrac{a^{2n-n} \cdot \cancel{r^{n(n-1)}} \cdot (r^n-1)^n}{(r-1)^n \cdot \cancel{r^{(n-1)\cdot n}}} = \dfrac{a^n\cdot(r^n-1)^n}{(r-1)^n}$

$\Rightarrow \dfrac{a^n\cdot(r^n-1)^n}{(r-1)^n} = \dfrac{a^n\cdot(r^n-1)^n}{(r-1)^n}$

LHS = RHS

(15) $a = p^{th}$ term $= A + (p-1).D$

$b = q^{th}$ term $= A + (q-1).D$

$c = r^{th}$ term $= A + (r-1).D$

$$\boxed{\begin{array}{l} \text{First term} = A \\ CD = D \end{array}}$$

To Prove: $(q-r).a + (r-p).b + (p-q).c = 0$

Proof: $(q-r).a = (q-r).[A + (p-1)D] = (q-r).A + (q-r).(p-1)D$

$(q-r).a = (q-r)A + (pq - q - rp + r)D$ —— (1)

Similarly $(r-p).b = (r-p).A + (rq - r - pq + p).D$ —— (2)

Similarly, $(p-q).c = (p-q).A + (pr - p - qr + q)D$ —— (3)

Add $\longrightarrow$ +

$\Rightarrow (q-r).a + (r-p).b + (p-q).c = (0).A + (0).D = 0 = RHS.$

Q. 16 If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP, then prove that a, b, c are in AP.

Given: $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right) \rightarrow AP$

$\Rightarrow a\left(\frac{c+b}{bc}\right), b\left(\frac{a+c}{ca}\right), c\left(\frac{b+a}{ab}\right) \longrightarrow AP.$

$\Rightarrow \frac{ac+ab}{bc}, \frac{ab+bc}{ca}, \frac{bc+ca}{ab} \longrightarrow AP.$

Add +1 in all the terms

$\Rightarrow \frac{ac+ab}{bc}+1, \frac{ab+bc}{ca}+1, \frac{bc+ca}{ab}+1 \longrightarrow AP.$

$\Rightarrow \frac{ac+ab+bc}{bc}, \frac{ab+bc+ca}{ca}, \frac{bc+ca+ab}{ab} \longrightarrow AP$

Divide $\boxed{K=(ab+bc+ca)}$ in all the terms

$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab} \longrightarrow AP.$

multiply $\boxed{K=abc}$ in each term

$\Rightarrow \frac{a\cancel{bc}}{\cancel{bc}}, \frac{\cancel{a}b\cancel{c}}{\cancel{ca}}, \frac{\cancel{ab}c}{\cancel{ab}} \rightarrow AP$

$\Rightarrow a, b, c \longrightarrow AP.$

Note:

$a_1, a_2, a_3, \dots a_n \rightarrow AP$

$\left.\begin{matrix}+K\\-K\\\times K\\\div K\end{matrix}\right\} \rightarrow$ New AP

a, b, c

Q.17 $a, b, c, d \to GP.$ Given

To prove: $(a^n + b^n), (b^n + c^n), (c^n + d^n) \to GP.$

Proof: $a, b, c, d \to G.P$

$a \quad ar \quad ar^2 \quad ar^3$

$$\boxed{\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n}}$$

$$\Rightarrow \frac{(ar)^n + (ar^2)^n}{(a)^n + (ar)^n} = \frac{(ar^2)^n + (ar^3)^n}{(ar)^n + (ar^2)^n}$$

$$\Rightarrow \frac{a^n \cdot r^n + a^n \cdot r^{2n}}{a^n + a^n \cdot r^n} = \frac{a^n \cdot r^{2n} + a^n \cdot r^{3n}}{a^n \cdot r^n + a^n \cdot r^{2n}}$$

$$\Rightarrow \frac{\cancel{a^n} r^n \cancel{(1 + r^n)}}{\cancel{a^n} \cancel{(1 + r^n)}} = \frac{\cancel{a^n} \cdot r^{2n} \cancel{(1 + r^n)}}{\cancel{a^n} \cdot r^n \cancel{(1 + r^n)}}$$

$$\Rightarrow r^n = \frac{r^{2n}}{r^n}$$

$$\Rightarrow r^n = r^{2n - n}$$

$$r^n = r^n$$

LHS = RHS

$$\frac{b^n + c^n}{a^n + b^n} = \frac{c^n + d^n}{b^n + c^n}$$

$\Rightarrow a^n + b^n, b^n + c^n, c^n + d^n \to GP$

Q.18

$x^2 - 3x + p = 0 \to a, b$

$x^2 - 12x + q = 0 \to c, d$

To Prove

$\boxed{(q+p) : (q-p) = 17 : 15}$

$a, b, c, d \longrightarrow$ G.P.

a, ar, ar^2, ar^3

$x^2 - 3x + p = 0 \to a, b$

$a + b = 3$ ✓

$ab = p$ ✓

$x^2 - 12x + q = 0 \to c, d$

$c + d = 12$ ✓

$cd = q$ ✓

$a + b = 3 \Rightarrow a + ar = 3 \Rightarrow a(1+r) = 3$ — (1)

$c + d = 12 \Rightarrow ar^2 + ar^3 = 12 \Rightarrow ar^2(1+r) = 12$ — (2)

Divide.

$\frac{1}{r^2} = \frac{3}{12} \Rightarrow \frac{1}{r^2} = \frac{1}{4} \Rightarrow r^2 = 4 \Rightarrow \boxed{r = \pm 2}$

$\because a + b = 3 \Rightarrow a + ar = 3 \Rightarrow \boxed{a(1+r) = 3} \Rightarrow \boxed{a = \frac{3}{1+r}}$

$\boxed{r = +2}$

$a = \frac{3}{1+r} = \frac{3}{1+2}$

$\boxed{a = 1}$

$\boxed{r = -2}$

$a = \frac{3}{1+r} = \frac{3}{1-2} = \frac{3}{-1}$

$\boxed{a = -3}$

LHS $= (q+p):(q-p)$ RHS $= 17:15$

$p = ab \quad q = cd$

$p = a \times ar \quad q = (ar^2).(ar^3)$

$p = a^2 r \quad q = a^2 r^5$

$r^2 = 4$

$\boxed{r^4 = 16}$

LHS $= (a^2r^5 + a^2r) : (a^2r^5 - a^2r)$

$$= \frac{\cancel{a^2r}(r^4+1)}{\cancel{a^2r}(r^4-1)} = \frac{r^4+1}{r^4-1} = \frac{16+1}{16-1}$$

$$= \frac{17}{15} = 17:15 = \text{RHS}.$$

$$\boxed{(q+p):(q-p) = 17:15}$$

Q.19 Two positive No. $= a, b$

$AM = \frac{a+b}{2}$

$GM = \sqrt{ab}$

$$\boxed{\frac{AM}{GM} = \frac{m}{n}}$$

To Prove:

$a:b = (m+\sqrt{m^2-n^2}) : (m-\sqrt{m^2-n^2})$

$$\frac{a}{b} = \frac{(m+\sqrt{m^2-n^2})}{m-\sqrt{m^2-n^2}} \times \frac{(m+\sqrt{m^2-n^2})}{m+\sqrt{m^2-n^2}}$$

$$\frac{a}{b} = \frac{m^2+m^2-n^2+2m\sqrt{m^2-n^2}}{\cancel{m^2}-(\cancel{m^2}-n^2)}$$

$$n = \frac{a}{b} = \frac{2m^2-n^2+2m\sqrt{m^2-n^2}}{n^2}$$

Given $\frac{AM}{GM} = \frac{m}{n}$

$$\Rightarrow \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

Square

$$\Rightarrow \frac{(a+b)^2}{(2\sqrt{ab})^2} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{a^2+b^2+2ab}{4 \cdot ab} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} + 2 = \frac{4m^2}{n^2}$$

$\frac{a}{b} = x$ (let) $x = ?$

$$\Rightarrow \frac{x}{1} + \frac{1}{x} + \frac{2}{1} = \frac{4m^2}{n^2}$$

$$\Rightarrow \frac{x^2+1+2x}{x} = \frac{4m^2}{n^2}$$

$$\Rightarrow \underbrace{n^2x^2} + \underbrace{n^2} + 2x.n^2 = x.4m^2$$

$$\Rightarrow n^2x^2 + 2x.n^2 - x.4m^2 + n^2 = 0$$

$$\Rightarrow n^2.x^2 + x(2n^2 - 4m^2) + n^2 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-2n^2 + 4m^2 \pm \sqrt{(2n^2-4m^2)^2 - 4.n^4}}{2n^2}$$

$$x = \frac{-2n^2 + 4m^2 \pm \sqrt{\cancel{4n^4} + 16m^4 - 16n^2m^2 - \cancel{4n^4}}}{2n^2}$$

$16\ m^2$

$$x = \frac{-2n^2 + 4m^2 \pm 4m\sqrt{m^2 - n^2}}{2n^2}$$

$$\frac{a}{b} = \boxed{x = \frac{-n^2 + 2m^2 \pm 2m\sqrt{m^2-n^2}}{n^2}}$$

Q.20

$a, b, c \rightarrow AP \rightarrow \boxed{b = \frac{a+c}{2}}$ — ①

$b, c, d \rightarrow GP \rightarrow \boxed{c^2 = bd}$ — ②

$\frac{1}{c}, \frac{1}{d}, \frac{1}{e} \rightarrow AP$

$\rightarrow \frac{1}{d} = \frac{\frac{1}{c}+\frac{1}{e}}{2}$

$\Downarrow$

$\Rightarrow \frac{1}{d} = \frac{\frac{e+c}{ce}}{2}$

$\Rightarrow \frac{1}{d} = \frac{e+c}{2ce}$

$\Rightarrow \boxed{d = \frac{2ce}{e+c}}$ — ③

By eqⁿ ②: $c^2 = bd$

$\Rightarrow c\cancel{^2} = \left(\frac{a+c}{\cancel{2}}\right) \cdot \left(\frac{\cancel{2}ce}{e+c}\right)$

$\Rightarrow$ ~~$c^2 = ace +$~~ .

To prove $\boxed{a, c, e \rightarrow GP.}$

$\rightarrow c^2 = ae$

$\Rightarrow c = \frac{(a+c).(e)}{(e+c)}$

$\Rightarrow \cancel{ec} + c^2 = ae + \cancel{ce}$

$\Rightarrow \boxed{c^2 = ae}$

$\therefore a, c, e$ are in GP.

Q.21 Find Sum of n-terms ?

(i) $5 + 55 + 555 + \cdots$

Sum of n-terms $= S_n = \underline{5} + \underline{55} + \underline{555} + \cdots$ n terms

$$\Rightarrow S_n = 5 \cdot (1 + 11 + 111 + \cdots \text{ n terms}) \times \frac{9}{9}$$ (self)

$$= \frac{5}{9} \cdot (\underline{9} + \underline{99} + \underline{999} + \cdots \text{ n-terms})$$

$$= \frac{5}{9} \cdot ([10-1] + [100-1] + [1000-1] + \cdots \text{ n-terms})$$

$$= \frac{5}{9} \left(\underbrace{[10 + 100 + 1000 + \cdots \text{ n-terms}]}_{a=10,\ r=10} - [1 + 1 + 1 \cdots \text{ n-terms}] \right)$$

G.P. $\rightarrow S = \dfrac{a(r^n - 1)}{r-1}$

$$= \frac{5}{9} \left\{ \left[\frac{10(10^n - 1)}{10-1} \right] - [n] \right\}$$

$$= \frac{50}{81} \cdot (10^n - 1) - \frac{5n}{9}$$ ✓

(ii) $0.6 + 0.66 + 0.666 + \dots$ n-terms

$$S_n = 0.6 + 0.66 + 0.666 + \dots \text{ n-terms}$$

$$S_n = 6 \cdot \{0.1 + 0.11 + 0.111 + \dots \text{ n-terms}\} \times \frac{9}{9}$$

$$= \frac{\cancel{6}^{2}}{\cancel{9}_{3}} \cdot \{0.9 + 0.99 + 0.999 + \dots \text{ n-terms}\}$$

$1-0.1$ $\quad 1-0.01$ $\quad 1-0.001$

$$= \frac{2}{3}\{(1-0.1) + (1-0.01) + (1-0.001) + \dots \text{ n-terms}\}$$

$$= \frac{2}{3}\{(1+1+1+\dots \text{ n-terms}) - (\underbrace{0.1 + 0.01 + 0.001 + \dots \text{ n-terms}}_{\text{G.P.}})\}$$

$$= \frac{2}{3}\{n - (\underbrace{\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{ n-terms}}_{\text{G.P.}})\}$$

G.P. $a = \frac{1}{10}$, $r = \frac{1}{10}$ $\quad S = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}$ $\quad r < 1$

$$= \frac{2}{3}\left\{n - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{\left(1-\frac{1}{10}\right)}\right\}$$

$$= \frac{2}{3}\left\{n - \frac{\cancel{\frac{1}{10}}\cdot\left(1-\frac{1}{10^n}\right)}{\frac{9}{\cancel{10}}}\right\} = \frac{2n}{3} - \frac{2\left(1-\frac{1}{10^n}\right)}{27}$$

Q.22 20^{th} term of series

$\underbrace{2 \times 4}_{a_1} + \underbrace{4 \times 6}_{a_2} + \underbrace{6 \times 8}_{a_3} + \dots \underbrace{n \text{ terms}}_{a_n}$

$a_n \rightarrow (\triangle) \times (\ )$

$a_{20} = ?$

$\underset{2\times1}{2}, \underset{2\times2}{4}, \underset{2\times3}{6}, \dots$

Similarly n^{th} term $= (2n) \times (2n+2) = a_n$

$n = 20$

$a_{20} = (2\times20) \times (2\times20+2)$

$$\boxed{a_{20} = 40 \times 42}$$

$$\boxed{a_{20} = 1680}$$

Q. 23

$? = S_n = \overbrace{3 + 7 + 13 + 21 + 31 + \dots + a_n}^{n\text{-terms}}$

not in A.P.

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \dots a_n$

Difference $\rightarrow$ 4 6 8 10

AP

$a_n = ? \rightarrow S_n = ?$

$S_n = 3 + 7 + 13 + 21 + 31 + \dots + a_{n-1} + a_n$

$- \quad S_n = \quad 3 + 7 + 13 + 21 \dots + a_{n-2} + a_{n-1} + a_n$

$0 = 3 + \underbrace{4 + 6 + 8 + 10 + \dots + (\ ) + (\ )}_{} - a_n$

AP $\rightarrow$ Sum ?

$\frac{n}{2}(2a + (n-1).d)$

no. of terms $= (n-1)$

$a = 4$

$d = 2$

$\Rightarrow a_n = 3 + \boxed{\frac{(n-1)}{\cancel{2}} \cdot [\cancel{2} \times 4 + (n-2).\cancel{2}]}$

$\Rightarrow a_n = 3 + \frac{(n-1)}{\cancel{2}} \cdot (\cancel{8}\ 4 + n - 2)$

$\Rightarrow a_n = 3 + (n-1) \cdot (n+2) = 3 + (n^2 + n - 2)$

$$\boxed{a_n = n^2 + n + 1}$$

$$\boxed{a_n = n^2 + n + 1}$$

Some Special series :

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

$$S_n = \sum_{K=1}^{n} a_K$$

$$S_n = \sum_{K=1}^{n} (K^2 + K + 1)$$

$$S_n = \left(\sum_{K=1}^{n} K^2\right) + \left(\sum_{K=1}^{n} K\right) + \left(\sum_{K=1}^{n} 1\right) \rightarrow \underbrace{1+1+1+\cdots+1}_{n\text{-times}}$$

$$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$$

$$S_n = n\left\{\frac{2n^2+3n+1}{6} + \frac{n+1}{2} + \frac{1}{1}\right\}$$

$$S_n = n\left\{\frac{2n^2+3n+1+3n+3+6}{6}\right\}$$

$$S_n = n\left\{\frac{\overset{n^2+3n+5}{\cancel{2n^2+6n+10}}}{\underset{3}{\cancel{6}}}\right\}$$

$$S_n = \frac{n(n^2+3n+5)}{3}$$

Q.24

Sum of first n- natural No. $= S_1 = 1+2+3+\cdots+n = \frac{n(n+1)}{2} = \sum_{k=1}^{n} k$

Sum of their ~~cubes~~ squares. $= S_2 = 1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} = \sum_{k=1}^{n} k^2$

Sum of their cubes. $= S_3 = 1^3+2^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \sum_{k=1}^{n} k^3$

<u>To Prove</u> $9S_2^2 = S_3(1+8.S_1)$

$$RHS = S_3 . (1+8.S_1)$$

$$= \left(\frac{n(n+1)}{2}\right)^2 . \left(1+\overset{4}{\cancel{8}}.\frac{n(n+1)}{\cancel{2}}\right)$$

$$= \left[\frac{n(n+1)}{2}\right]^2 . [\underbrace{1+4n^2+4n}_{(2n+1)^2}]$$

$$= \left[\frac{n(n+1)}{2}\right]^2 . (2n+1)^2 \quad = \left[\frac{n(n+1)(2n+1)}{2}\right]^2$$

$$LHS = 9(S_2)^2$$

$$= \underset{3^2}{9} . \left[\frac{n(n+1)(2n+1)}{6}\right]^2$$

$$= \left[\cancel{3}.\frac{n(n+1)(2n+1)}{\underset{2}{\cancel{6}}}\right]^2$$

$$= RHS.$$

Q.25 Sum of n-terms = ? = S_n

$$S_n = \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots + \boxed{a_n}$$

a_1, a_2, a_3

odd $\begin{cases} 2n-1 & (n=3) = 5 \\ 2n+1 & = 7 \end{cases}$

$$n^{th}\text{ term} = a_n = \frac{1^3+2^3+3^3+\dots+n^3}{1+3+5+\dots+(2n-1)} \Rightarrow a_n = \frac{\left[\frac{n(n+1)}{2}\right]^2}{\frac{n}{\cancel{2}}\left[\cancel{2}+(n-1).\cancel{2}\right]}$$

AP, no. of terms $= n$, $a = 1$, $d = 2$

Sum $\rightarrow \frac{n}{2}(2a+(n-1).d)$

$$a_n = \frac{\frac{n^2.(n+1)^2}{2^2}}{n\,(\cancel{1}+n-\cancel{1})} = \frac{\frac{\cancel{n^2}.(n+1)^2}{2^2}}{\cancel{n^2}} = \frac{(n+1)^2}{4} = \frac{n^2+2n+1}{4} = \frac{n^2}{4}+\frac{n}{2}+\frac{1}{4}$$

$$\boxed{a_n = \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}}$$

$$\boxed{a_n = \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}}$$

$$S_n = \sum_{K=1}^{n} (a_K) \;\cancel{=}\; \Sigma$$

$$S_n = \sum_{K=1}^{n} \left(\frac{K^2}{4} + \frac{K}{2} + \frac{1}{4}\right)$$

$$S_n = \Sigma \frac{K^2}{4} + \Sigma \frac{K}{2} + \Sigma\left(\frac{1}{4} \times 1\right)$$

$$S_n = \frac{1}{4} \sum_{K=1}^{n} K^2 + \frac{1}{2} \sum_{K=1}^{n} K + \frac{1}{4} \times \sum_{K=1}^{n} 1$$

$$S_n = \frac{1}{4} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \cdot \left(\frac{n(n+1)}{2}\right) + \frac{1}{4}(n)$$

Q.26 Show that $\dfrac{1.2^2+2.3^2+\dots+n.(n+1)^2}{1^2\times 2+2^2.3+\dots+n^2.(n+1)} = \dfrac{3n+5}{3n+1}$

$\dfrac{S_n^N}{S_n^D}$

Numerator

$S_n^N = \underbrace{1.2^2}_{a_1} + \underbrace{2.3^2}_{a_2} + \dots + \underbrace{n(n+1)^2}_{a_n}$

$a_n = n(n+1)^2 = n(n^2+2n+1)$

$a_n = n^3+2n^2+n$ ✓

$S_n^N = \sum_{K=1}^{n} a_K = \sum_{K=1}^{n}(K^3+2K^2+K)$

$= \sum_{K=1}^{n} K^3 + 2\sum_{K=1}^{n} K^2 + \sum_{K=1}^{n} K$

$= \left[\dfrac{n(n+1)}{2}\right]^2 + \cancel{2}.\dfrac{n(n+1)(2n+1)}{\cancel{6}\,3} + \dfrac{n(n+1)}{2}$

$n(n+1) = n^2+n$

$= n(n+1)\left\{\dfrac{n(n+1)}{4} + \dfrac{2n+1}{3} + \dfrac{1}{2}\right\}$

$= n(n+1).\left\{\dfrac{3n^2+3n+8n+4+6}{12}\right\}$

$= \dfrac{n(n+1)(3n^2+11n+10)}{12}$

$= n(n+1).\dfrac{(3n^2+6n+5n+10)}{12}$

$S_n^N = \dfrac{n(n+1).(3n+5).(n+2)}{12}$ ✓

$$S_n^D = \underbrace{1^2 \cdot 2}_{a_1} + \underbrace{2^2 \cdot 3}_{a_2} + \cdots + \underbrace{n^2 \cdot (n+1)}_{a_n}$$

$$\boxed{a_n = n^2 \cdot (n+1)}$$

$$a_n = n^3 + n^2$$

$$S_n^D = \sum_{K=1}^{n} a_K = \sum_{K=1}^{n} K^3 + K^2$$

$$S_n^D = \sum_{K=1}^{n} K^3 + \sum_{K=1}^{n} K^2$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{} \left\{ \frac{n(n+1)}{4} + \frac{2n+1}{6} \right\}$$

$$= \frac{n(n+1)(3n^2+3n+4n+2)}{12}$$

$$= \frac{n(n+1)(3n^2+7n+2)}{12}$$

$$= \frac{n(n+1)(3n^2+6n+n+2)}{12}$$

$$S_n^D = \frac{n(n+1).(3n+1)(n+2)}{12}$$

$$\frac{S_n^N}{S_n^D} = \frac{\frac{\cancel{n}\cancel{(n+1)}(3n+5)\cancel{(n+2)}}{\cancel{12}}}{\frac{\cancel{n}\cancel{(n+1)}(3n+1).\cancel{(n+2)}}{\cancel{12}}}$$

$$= \frac{3n+5}{3n+1} = RHS.$$

$\frac{6000}{500} = 12$

$$\begin{array}{r} 72 \\ \times\ 65 \\ \hline 360 \\ 432\times \\ \hline 4680 \end{array}$$

Practical Problems (A.P., G.P.)

$\boxed{Q.27}$ Total Price $= 12000 = 6000 + 6000$

Cash: 6000

Instalments: ₹ 500 × 12 instalment

1st → 500 + 12% of 6000

2nd → 500 + 12% of 5500

3rd → 500 + 12% of 5000

⋮

12th → 500 + 12% of 500

Total amount of the tractor

$$= \underset{(cash)}{6000} + \left(500 + \frac{12}{100}\times 6000\right) + \left(500 + \frac{12}{100}\times 5500\right) + \cdots + \left(500 + \frac{12}{100}\times 500\right)$$

12-times

$$= 6000 + 500\times 12 + \frac{12}{100}\times\left(6000 + 5500 + \cdots + 500\right)$$

A.P. $n = 12$

$$= 6000 + 6000 + \frac{12}{100}\times\frac{n}{2}\times(a+l)$$

$$= 6000 + 6000 + \frac{12}{100}\times\frac{\cancel{12}^{6}}{\cancel{2}}\times(500+6000)$$

$$= 12000 + \frac{12\times 6\times 65\cancel{00}}{\cancel{100}} = 12000 + 4680$$

$$= ₹16680$$

Q. 28 Total original Price $= ₹22000$ → ₹4000 cash; ₹18000 Installments (₹1000 × 18 installments)

1st → $1000 + 10\% \text{ of } 18000$

2nd → $1000 + 10\% \text{ of } 17000$

3rd → $1000 + 10\% \text{ of } 16000$

⋮

18th → $1000 + 10\% \text{ of } 1000$

Total $\quad 1000 \times 18 + \frac{10}{100} \text{ of } \underbrace{(18000 + 17000 + \dots + 1000)}_{\text{A.P.}\ n = 18}$

$$= 18000 + \frac{10}{100} \times \frac{n}{2}(a + l)$$

$$= 18000 + \frac{10}{100} \times \frac{18}{2}(1000 + 18000)$$

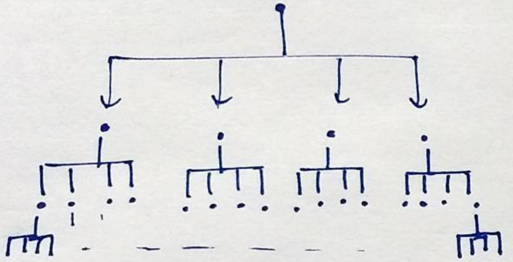
$$= 18000 + \frac{1}{10} \times 9 \times 19000 = 18000 + 17100$$

Total cost of Scooter $= 4000 + 18000 + 17100$

$$= ₹39100$$

Q.29

₹ 0.50 = 50 paise for one letter



1st set → $Cost_1 \to 1 \times 4 \times 0.50$ = ₹ 2 = a_1

2nd set → $Cost_2 \to 4 \times 4 \times 0.50$ = ₹ 8 = a_2

3rd set → $Cost_3 \to 4 \times 4 \times 4 \times 0.50$ = ₹ 32 = a_3

⋮

8th set → $Cost_8 \to$ = a_8

total cost

Series = ₹ 2 + ₹ 8 + ₹ 32 + --- + a_8 → G.P.

Sum of G.P. $= \dfrac{a(r^n - 1)}{r-1}$ | $a = 2$, $r = 4$, $n = 8$

total Cost $= \dfrac{2(4^8 - 1)}{4-1} = 2 \times \dfrac{\overset{21845}{\cancel{65535}}}{\cancel{3}}$

= ₹ 43690

Rough

$4^8 = 2^{16}$

$2^{12} = 4096$

$2^{16} = 2^{12} \times 2^4$

$= 4096 \times 16$

$= 65536$

Q.30 Deposited ₹10,000 | Rate = 5% (SI) | Amount = Principal + Interest

Principal

₹17000 ₹20000

₹10000 → • [1] • [2] [3] . . . [14] • [15] [16] . . . [20] • [21]

Amount = Principal + SI

= 10000 + 5% of 10000

$= 10000 + \frac{5}{\cancel{100}} \times \cancel{10000}$

= 10000 + 500

= ₹10500

After one year → Amount_1 = 10000 + 500

After two years → A_2 = 10000 + 500 + 500

After 3-years → A_3 = 10000 + 500 × 3

⋮

After 14-years → A_{14} = 10000 + 500 × 14 = 10000 + 7000 = 17000

Amount in 15th year

After 20-years → A_{20} = 10000 + 500 × 20 = 10000 + 10000

Q.31

Present cost = ₹ 15625 → 1 . 2 . 3 . 4 . 5 .

15625 12500 10000

Rate = −20 %

$\boxed{\text{Estimated value at the end of 5 years}}$ = ? = 5120

E.V.

EV after one year = 15625 − 20% of 15625 = $15625 - \frac{20}{100} \times 15625$
a_1

= ₹ 12500

E.V. after 2-years = 12500 − 20% of 12500
a_2

$= 12500 - \frac{20}{100} \times 12500$

= 12500 − 2500

= 10000

T_1 T_2 T_3 T_4 T_5 T_6

15625 | 1 year | 12500 | 2 yrs | 10000 | 3 yrs | | 4 yrs | | 5 yrs

a_0 a_1 a_2 a_3 a_4 a_5 = ?

$\frac{12500}{15625} = \frac{4}{5} = \frac{10000}{12500}$

G.P. → $a = 15625$
$r = 4/5$,

$a_5 = T_6$
$= a \cdot r^{n-1}$
$= 15625 \cdot \left(\frac{4}{5}\right)^5$
$= 5120$

Q.32

$\boxed{\begin{array}{l}1 \text{ worker} \\ 1 \text{ Day}\end{array}} \rightarrow$ 1 work $=$ 1 chocolate

Ideal $\rightarrow$ $\overbrace{\boxed{150}\ \boxed{150}\ \boxed{150}\ \cdots\ \boxed{150}}^{n\text{-Days}}$ $\rightarrow$ $\boxed{\text{Total work} = 150 \cdot n}$ = total No. of choco.

Actual $\underbrace{\underbrace{\boxed{150}\ \boxed{146}\ \boxed{142}\ \cdots\ \boxed{\ }}_{n}\ \underbrace{\boxed{\ }\ \boxed{\ }\ \cdots\ \boxed{\ }}_{8 \text{ more Days}}}_{(n+8) \text{ Days}}$ $\Rightarrow$ total work $=$?

$\boxed{-2n^2 + 130n + 1088}$

In ~~the~~ actual case, total work $= \underbrace{150 + 146 + 142 + \cdots}_{\text{Sum}}$

A.P. $a = (150)$

$d = (-4)$

no. of terms $= (n+8)$

$$= \frac{(n+8)}{\cancel{2}}\left[\cancel{2}\times 150 + (n+7)\cdot(\cancel{-4})\right]$$

$$= (n+8)(150 - 2n - 14) = (n+8)(136 - 2n) = -2n^2 + 130n + 1088$$

total work

Ideal = actual

$\Rightarrow 150 \cdot n = -2n^2 + 120n + 1088$

$\Rightarrow 2n^2 + 30n - 1088 = 0$

$\Rightarrow \boxed{1 \cdot n^2 + 15n - 544 = 0}$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-15 \pm \sqrt{(15)^2 - 4 \cdot (1) \cdot (-544)}}{2 \times 1}$$

$$n = \frac{-15 \pm \sqrt{225 + 2176}}{2}$$

$$n = \frac{-15 \pm \sqrt{2401}}{2}$$

$\sqrt{2401} = 49$

$$n = \frac{-15 \pm 49}{2}$$

$(+)$

$$n = \frac{-15 + 49}{2}$$

$$n = \frac{34}{2}$$

$n = 17$

$(-)$

$$n = \frac{-15 - 49}{2}$$

$n = (-)$

no. of Days in which the work was completed. actual

$= (n + 8)$

$= 17 + 8$

$= 25$ Days